Graphene ratchets

arXiv:1103.5597

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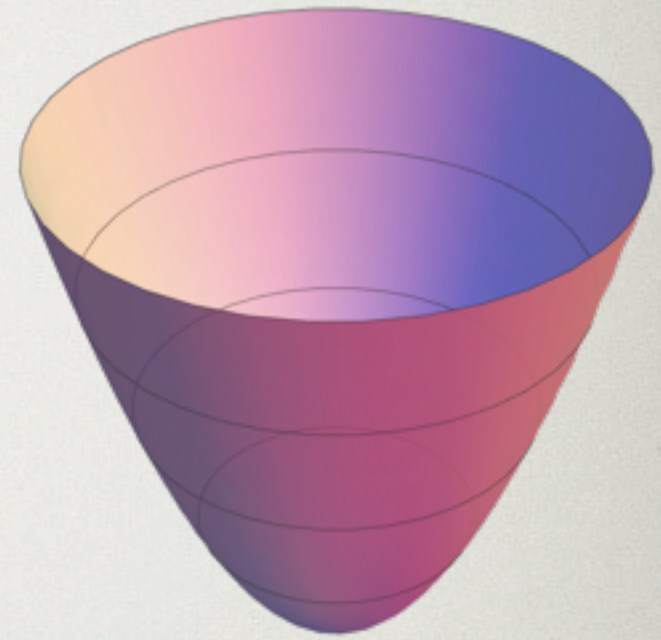
Outline

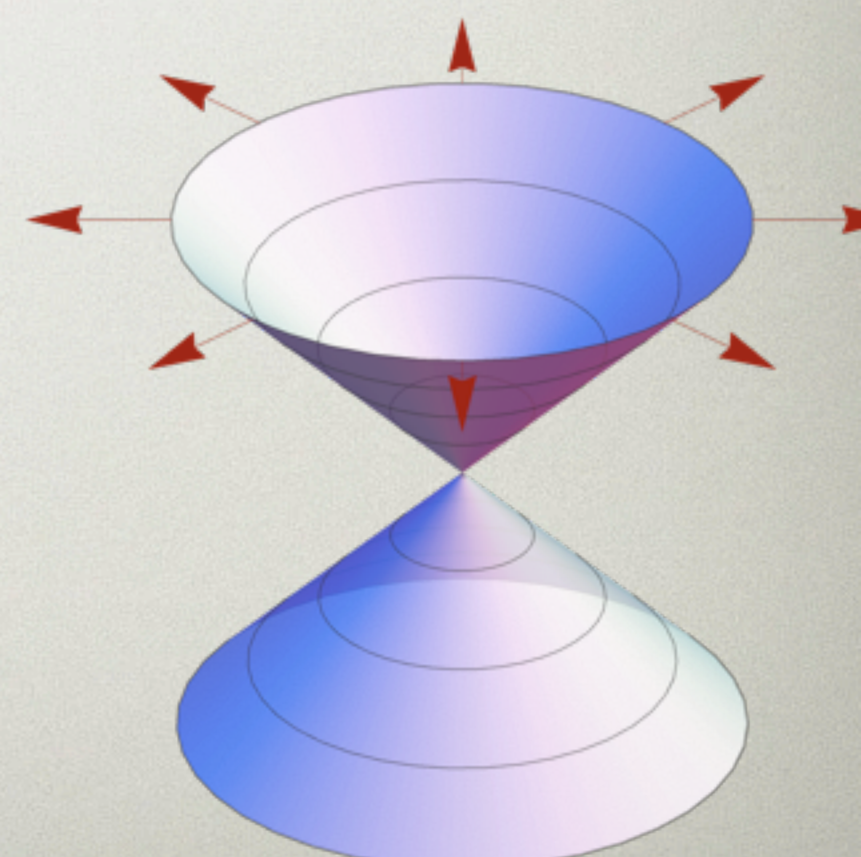
- Electrons, chirality and transport
- Adiabatic quantum pumping
- Non-adiabatic ratchets
  - Floquet theory
  - Pumping response
  - Rectification, pumping - transmission correspondence, semiclassical limit
- Conclusions

Electrons in graphene

- 2D electron gas $$H = \frac{k^2}{2m^*}$$
  - Velocity increases with k



- Graphene $$H = v_F \boldsymbol{k} \cdot \boldsymbol{\sigma}$$
  - Eigenstates are chiral
  - Velocity is k-independent

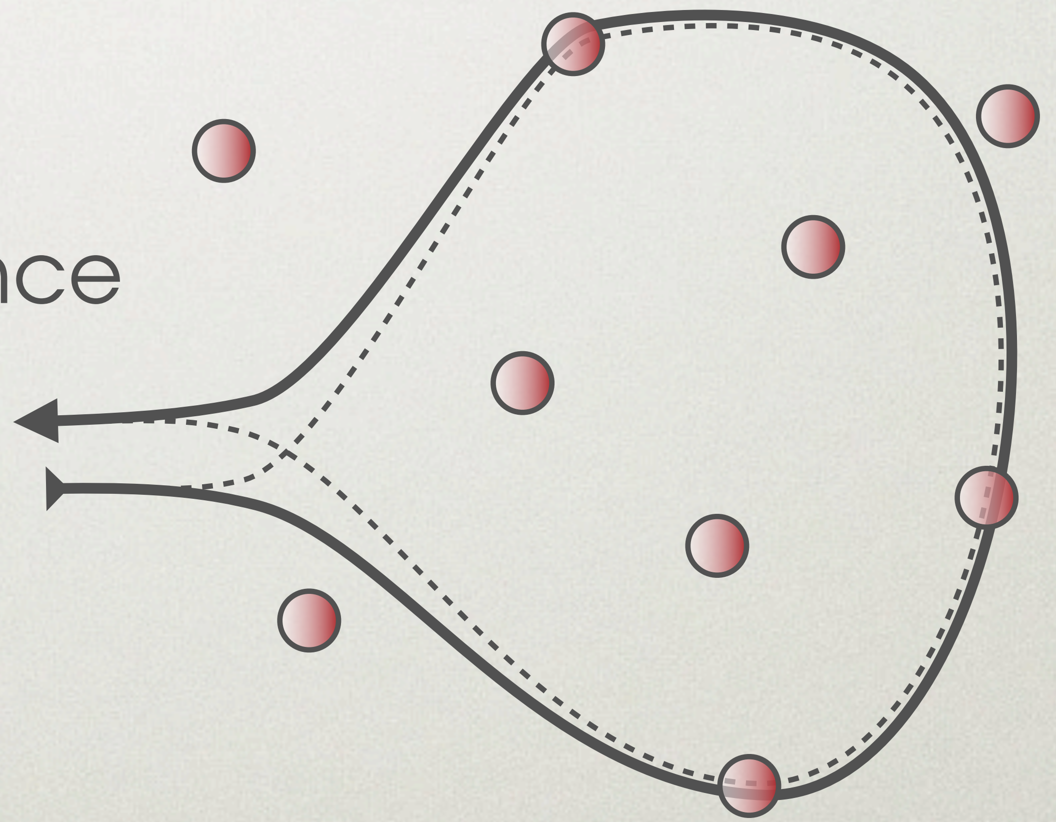


Chirality and transport

- Chirality tends to delocalize electrons
  - Weak antilocalization

Destructive interference

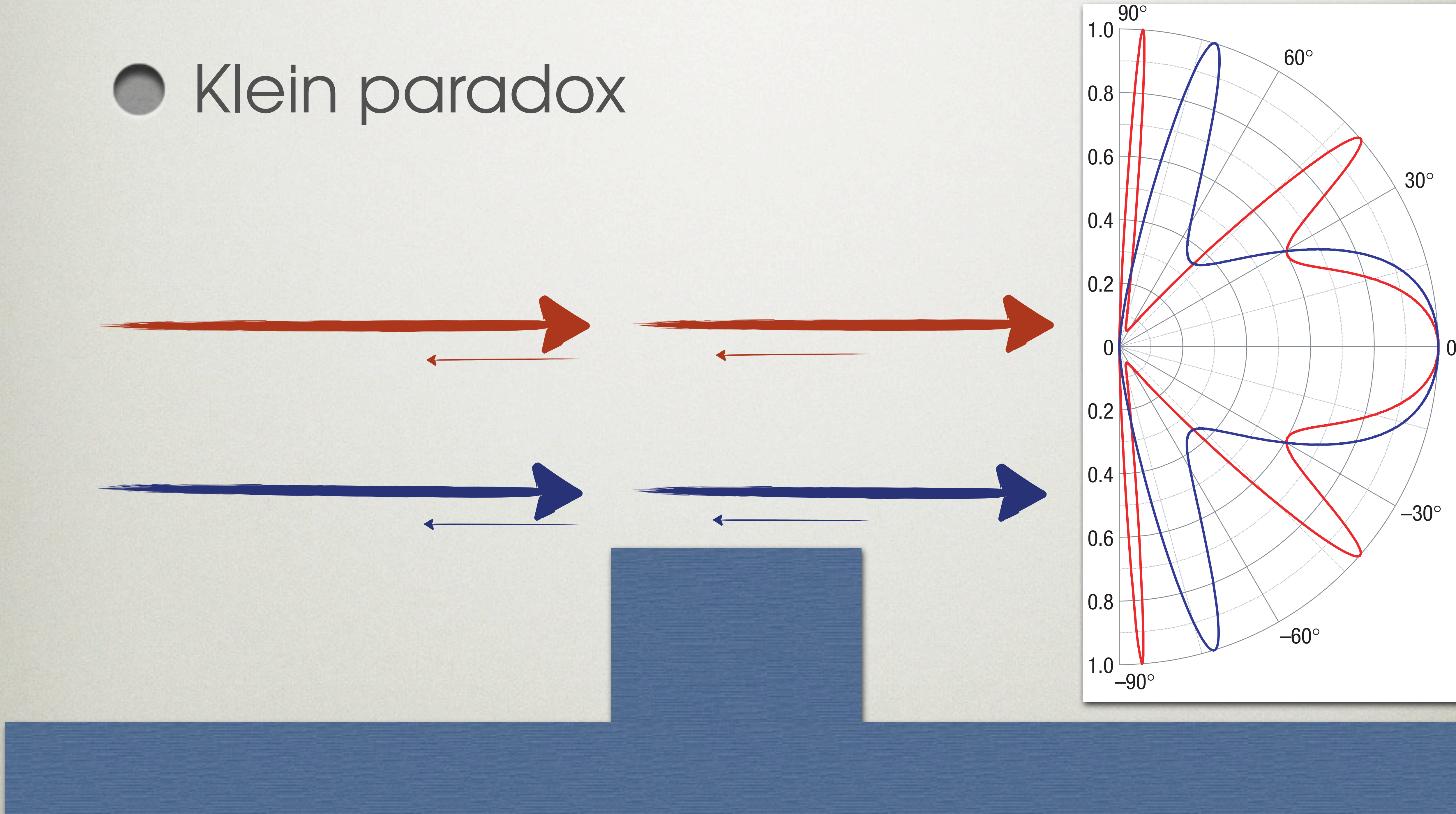
$$e^{\frac{i}{2}2\pi\sigma_z} = -\sigma_0$$



X. Wu, X. Li, Z. Song, C. Berger and W. A. de Heer, Phys. Rev. Lett. **98** 136801 (2007)

Chirality and transport

- Chirality tends to delocalize electrons
  - Klein paradox



M. Katsnelson,K. Novoselov and A. Geim. Nature Physics **2** 620--625 (2006)

Chirality and transport

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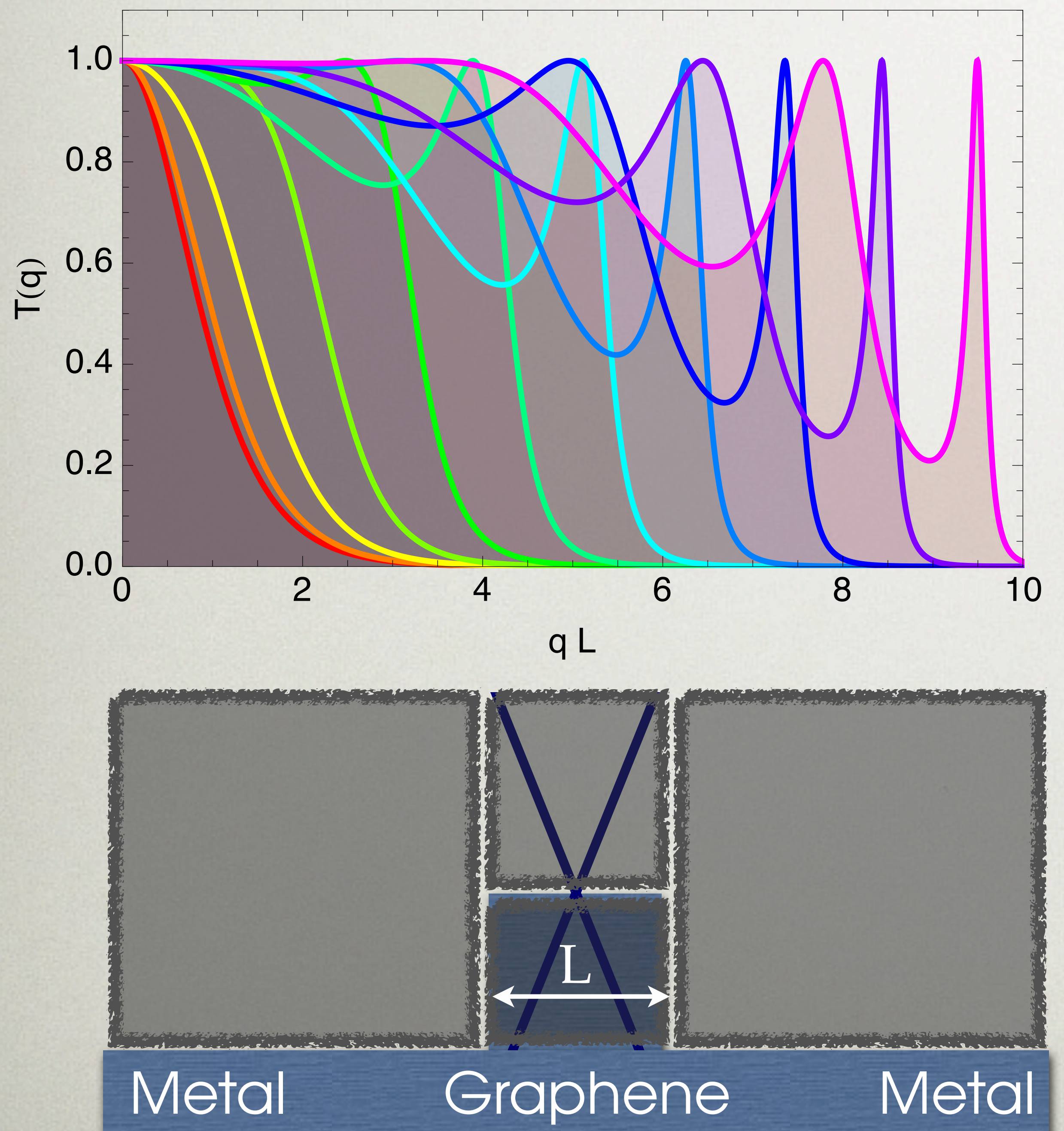
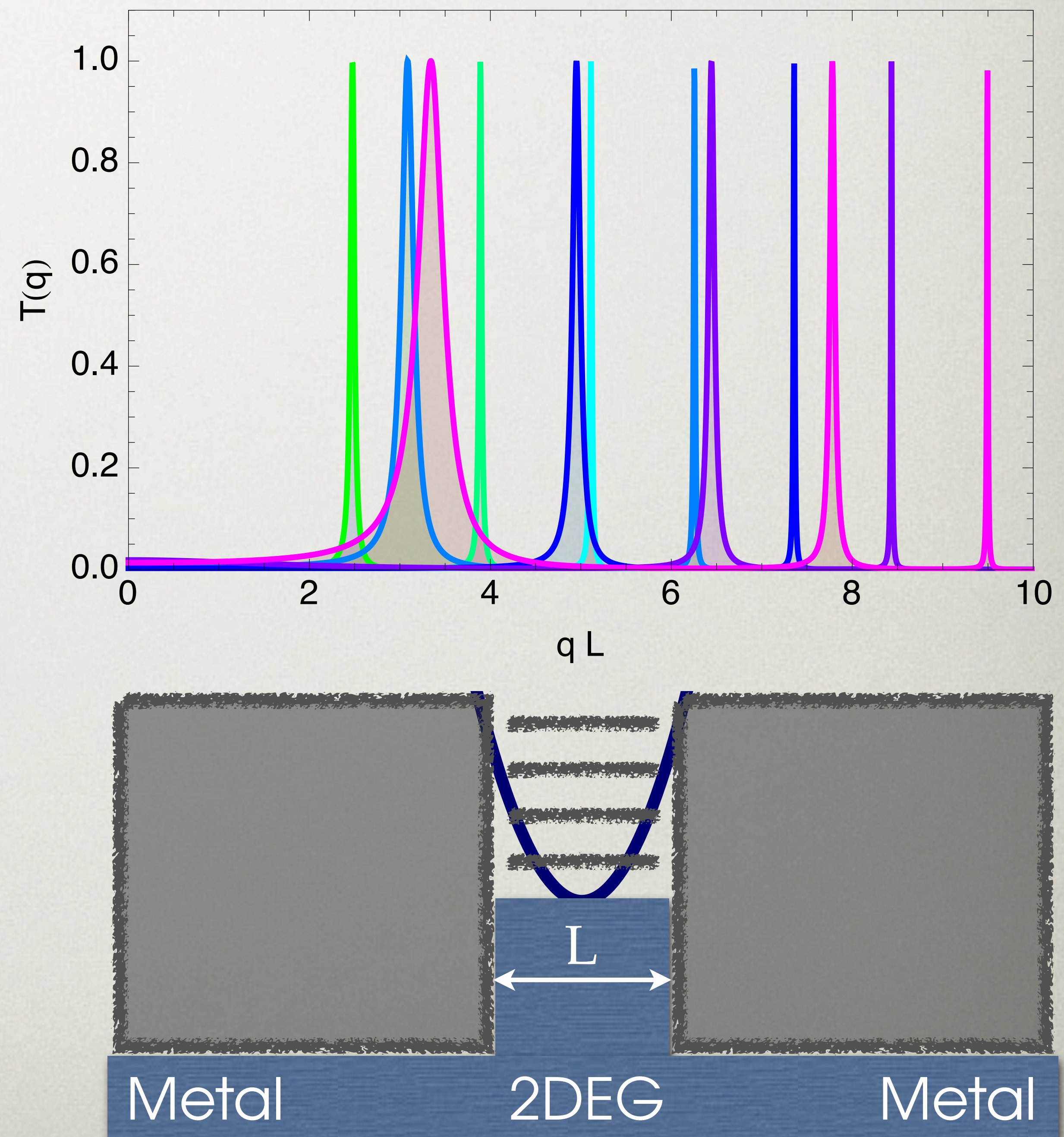
Graphene



Open system

2DEG



Closed system

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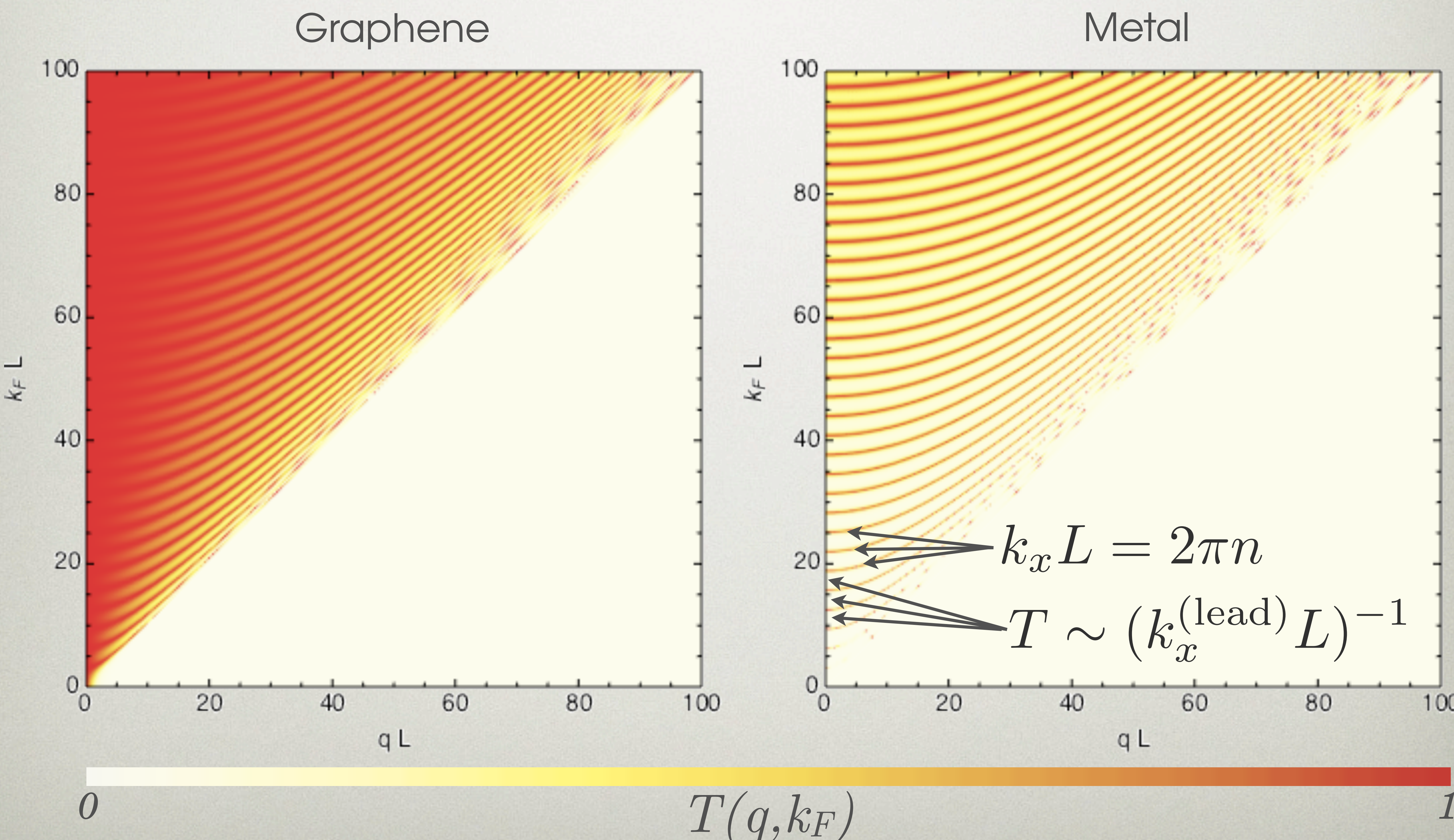


Quantum pumping

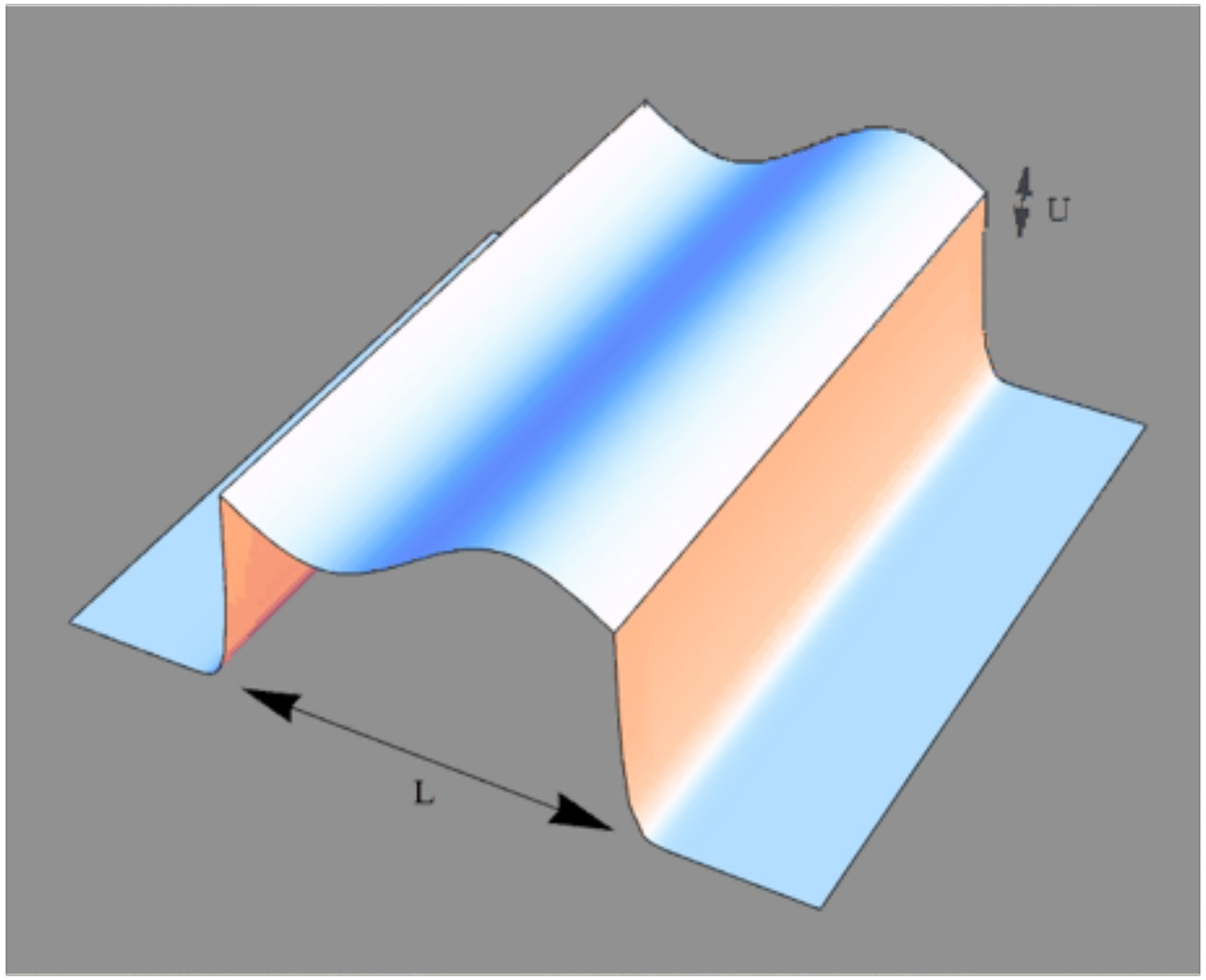
- How does chirality impact the response of electrons under local driving?

Level spacing

E_L

$$E_L^{(M)} = \frac{\hbar^2}{2m^* L^2}$$

$$E_L^{(G)} = \frac{\hbar v_F}{L}$$



A quantum pump

Frequency ω

Amplitude U

Pumping regimes

- Adiabatic limit $\omega \ll U, E_L$
  - Weak driving $U \ll E_L$
  - Strong driving $U \gg E_L$
- Non-adiabatic limit $U \ll \omega$
  - Weak driving $U \ll E_L$
  - Strong driving $U \gg E_L$

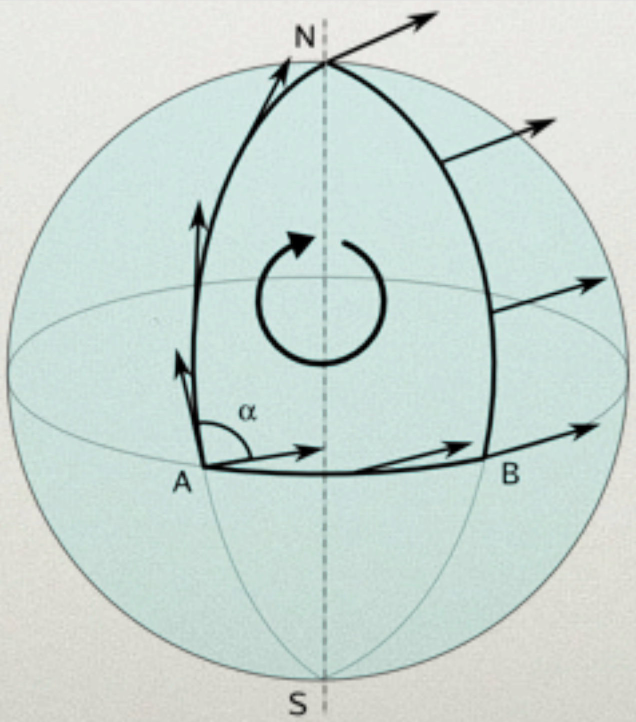
Adiabatic pumping

- Geometric formulation of pumping

Brouwer's formula

$$Q_{L\to R} = \frac{e}{\pi}\mathrm{Im}\oint d\vec{\chi}\,\langle s_L|\vec{\nabla}_\chi|s_L\rangle$$

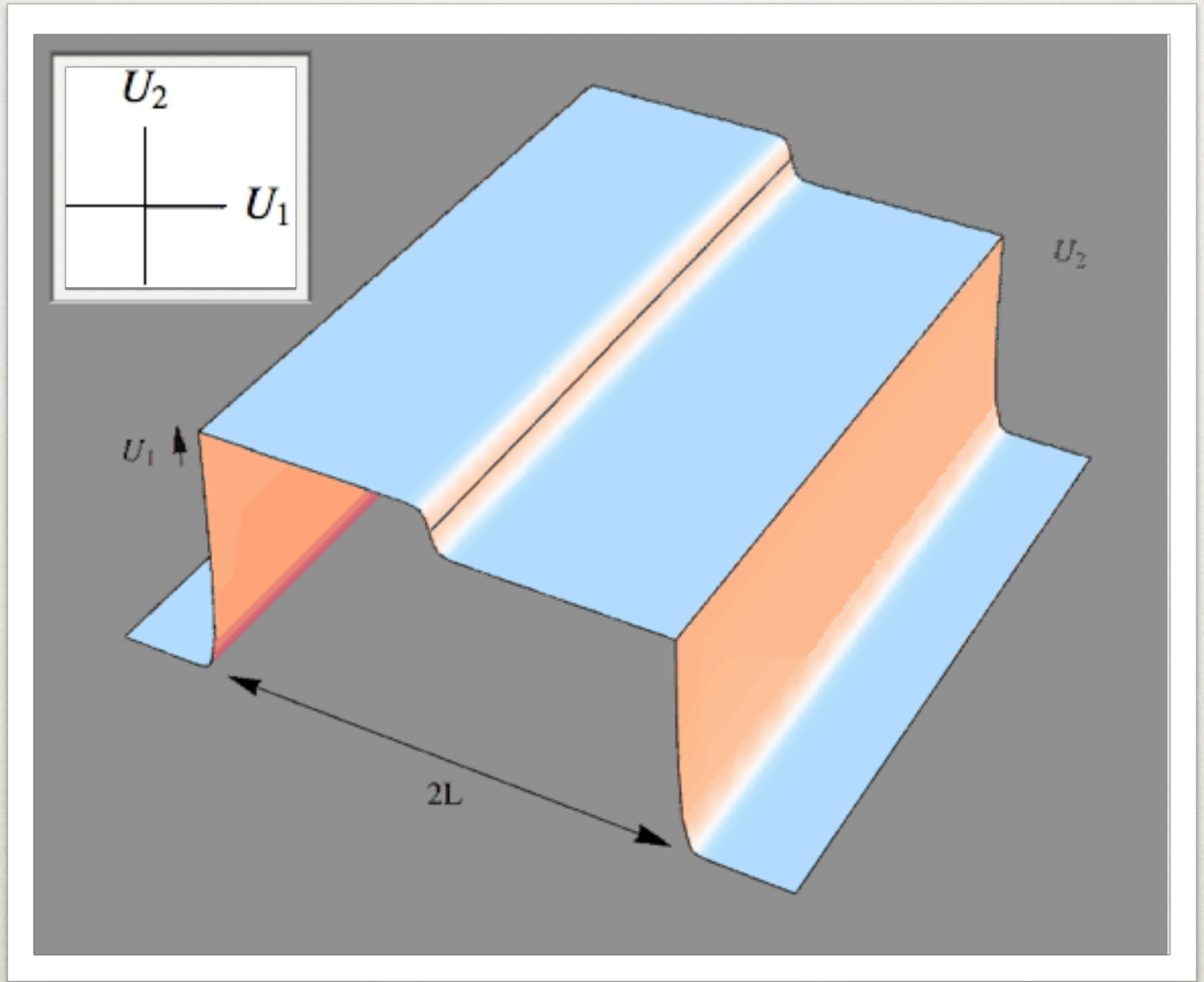
$$\langle i|s_L\rangle = S_{L,i}(\vec{\chi}) = (r, t')$$



P. Brouwer, Phys. Rev. B **58**, 10135(R) (1998)

Adiabatic pumping

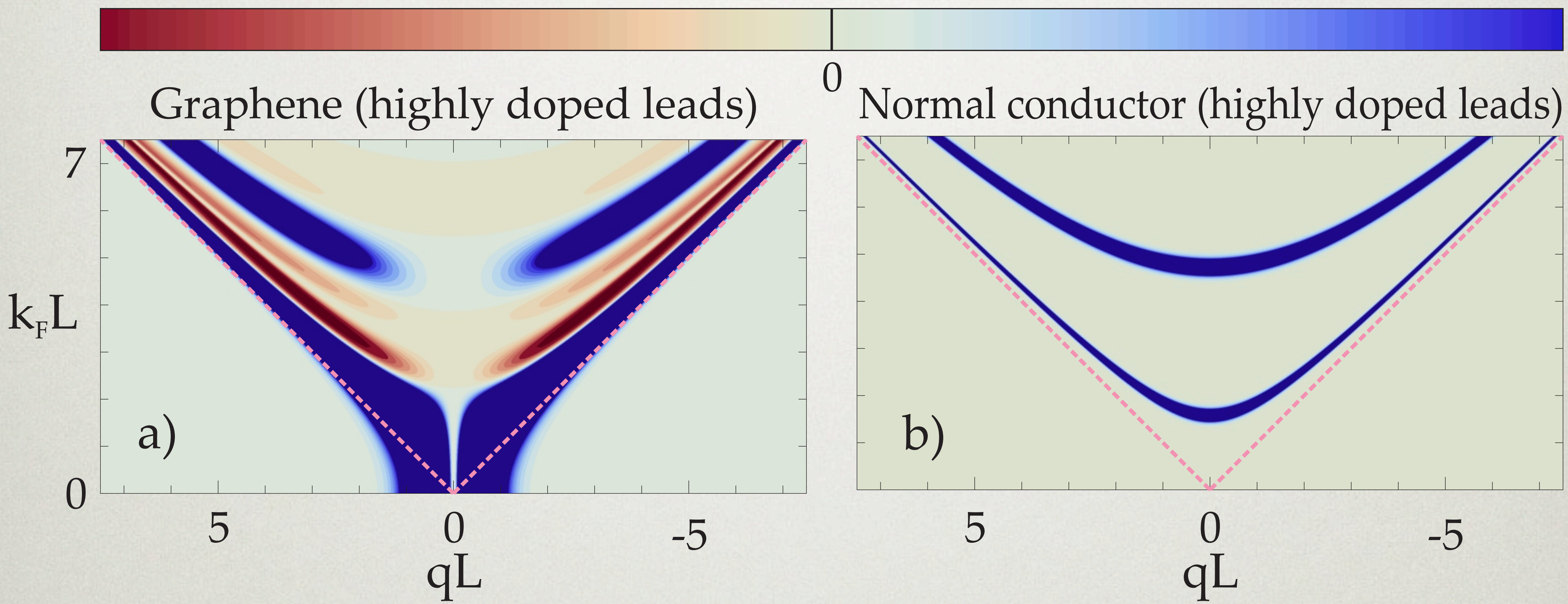
- Minimal setup: two parameter pumping



$$Q_{L\to R} = \frac{e}{\pi}\mathrm{Im}\iint dU_1 dU_2 \langle \partial_{U_1} s_L | \partial_{U_2} s_L \rangle$$

Adiabatic pumping

- Adiabatic pumping response $\frac{Q_{L\to R}}{\pi\,(U/E_L)^2}\frac{1}{N_p}$

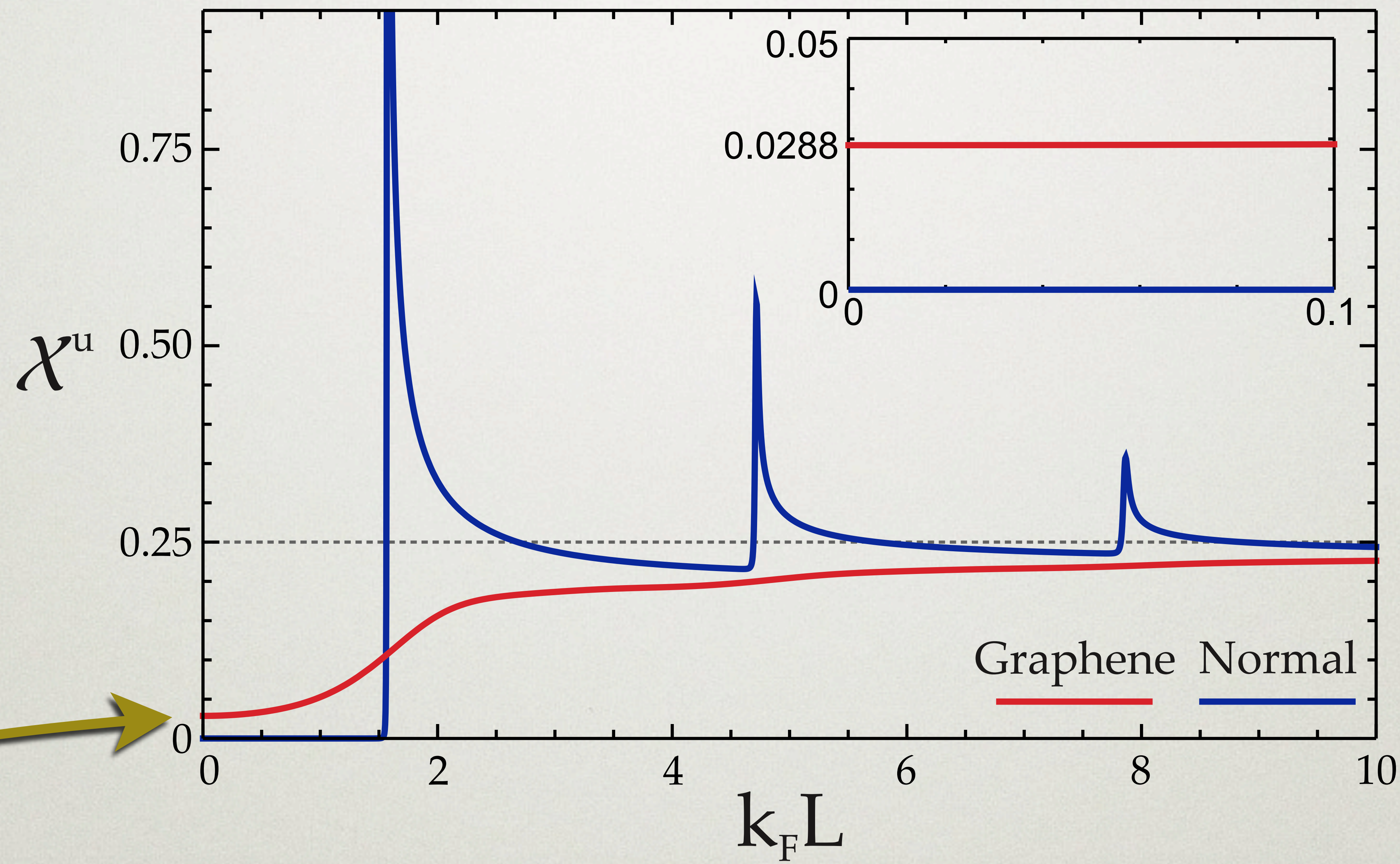


- Chirality enables pumping of evanescent modes close to $q=0$

E. Prada, P. San-Jose and H. Schomerus, Phys. Rev. B **80**, 245414 (2009).

Adiabatic pumping

- Total response: summing over modes



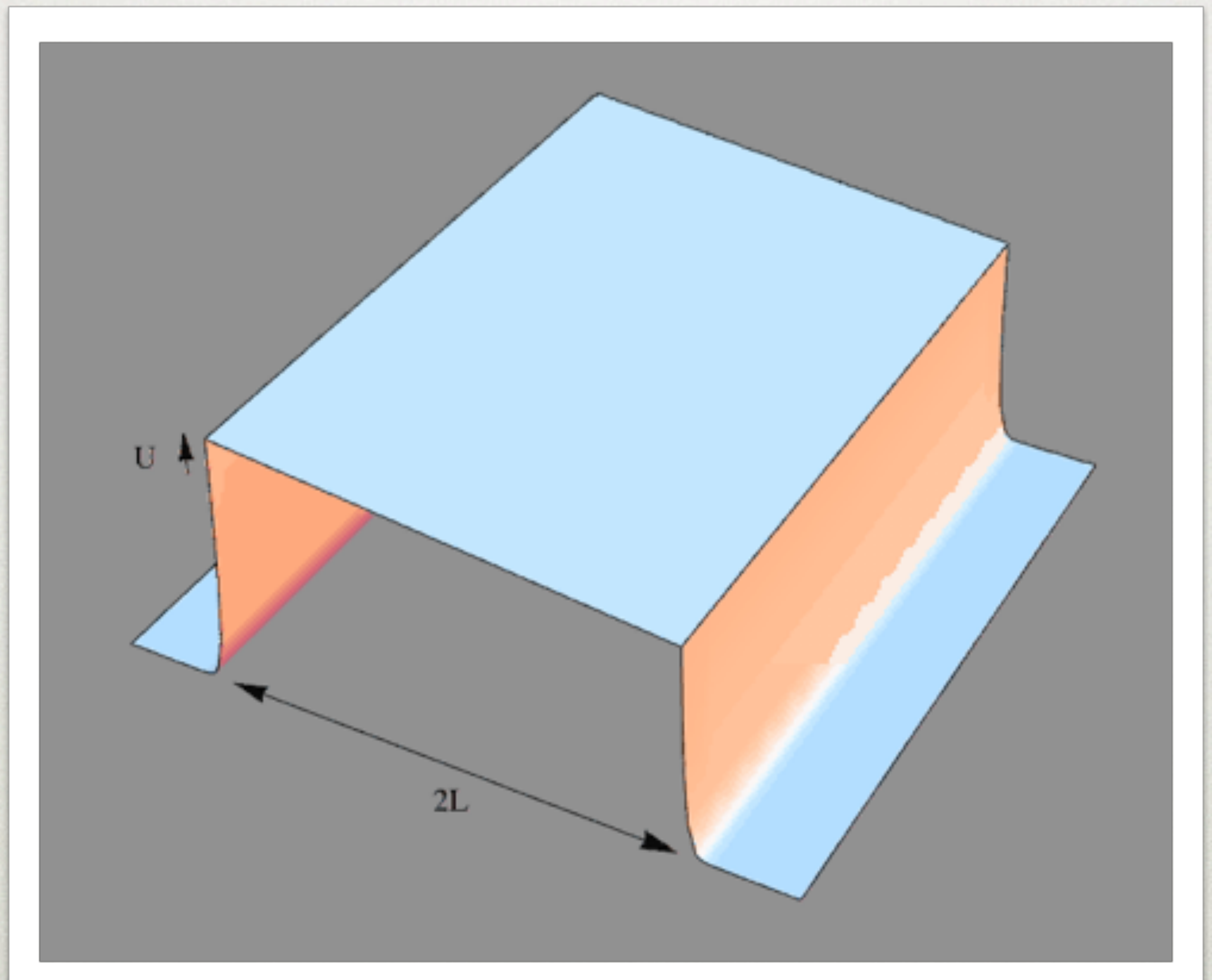
Universal response (W≫L)

$$\int_0^\infty dq \frac{\sinh^2(q)\,[2q\cosh(2q) - \sinh(2q)]}{\pi q^3 \cosh^4(2q)} = 0.0288$$

Non-adiabatic pumping

- Minimal pumping requirements:

single parameter driving + left-right asymmetry



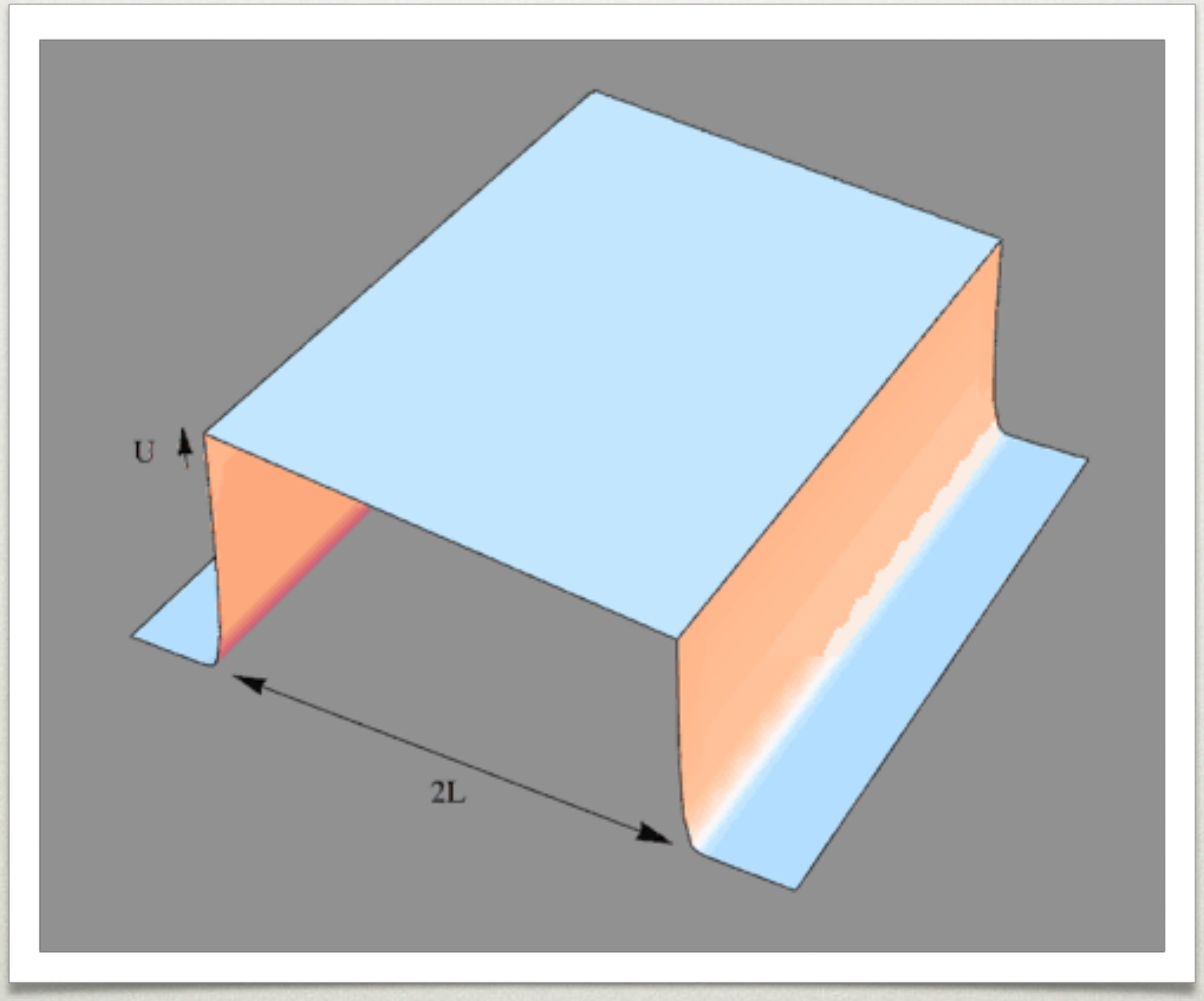
Minimal non-adiabatic pump

S. Kohler, J. Lehmann and P. Hänggi, Physics Reports **406** 379 - 443 (2005)

Non-adiabatic pumping

- Minimal pumping requirements:

single parameter driving + left-right asymmetry



Quantum ratchet

S. Kohler, J. Lehmann and P. Hänggi, Physics Reports **406** 379 - 443 (2005)

Non-adiabatic pumping

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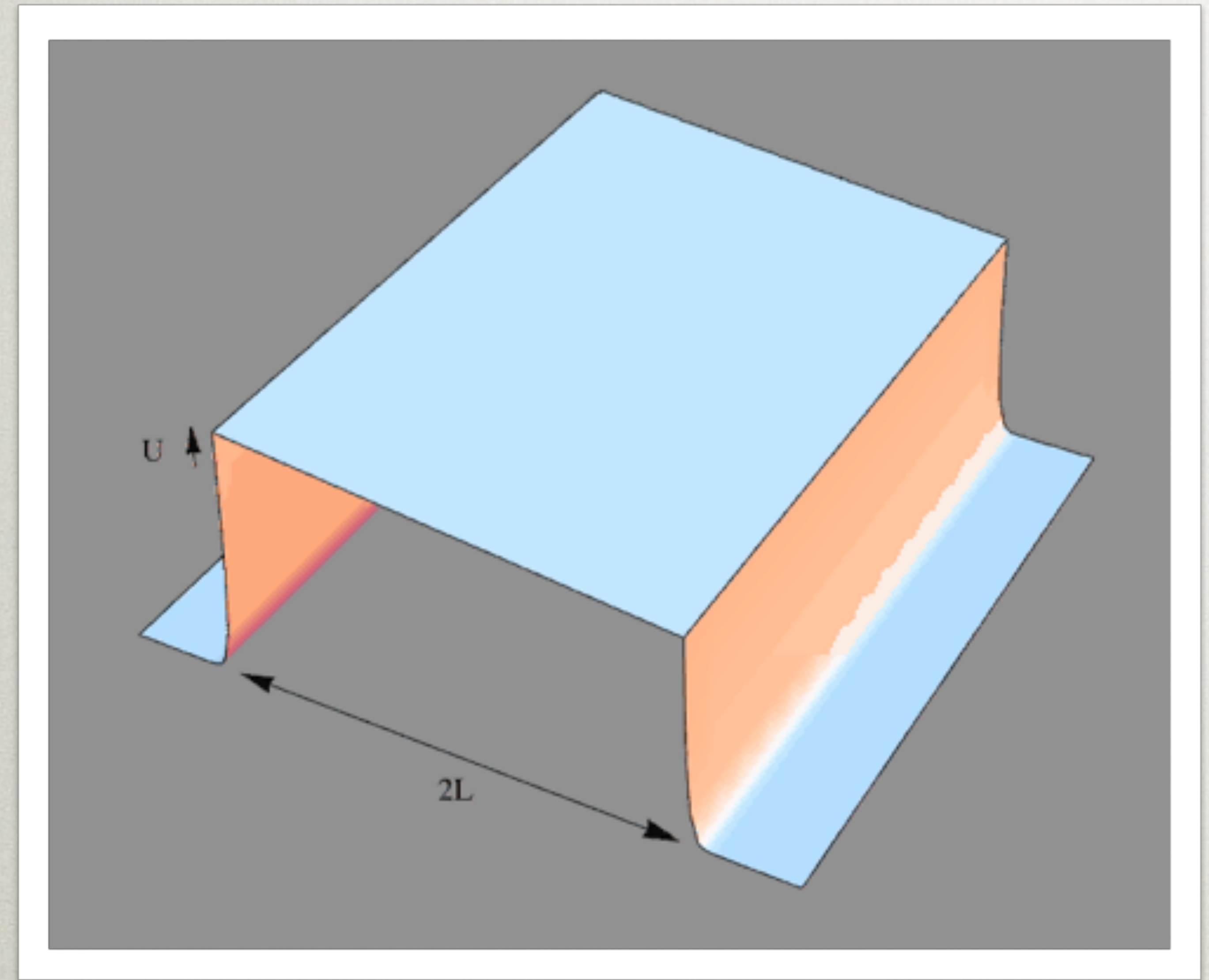

Quantum ratchet

Repeated ratchet units

S. Kohler, J. Lehmann and P. Hänggi, Physics Reports **406** 379 - 443 (2005)

Non-adiabatic pumping

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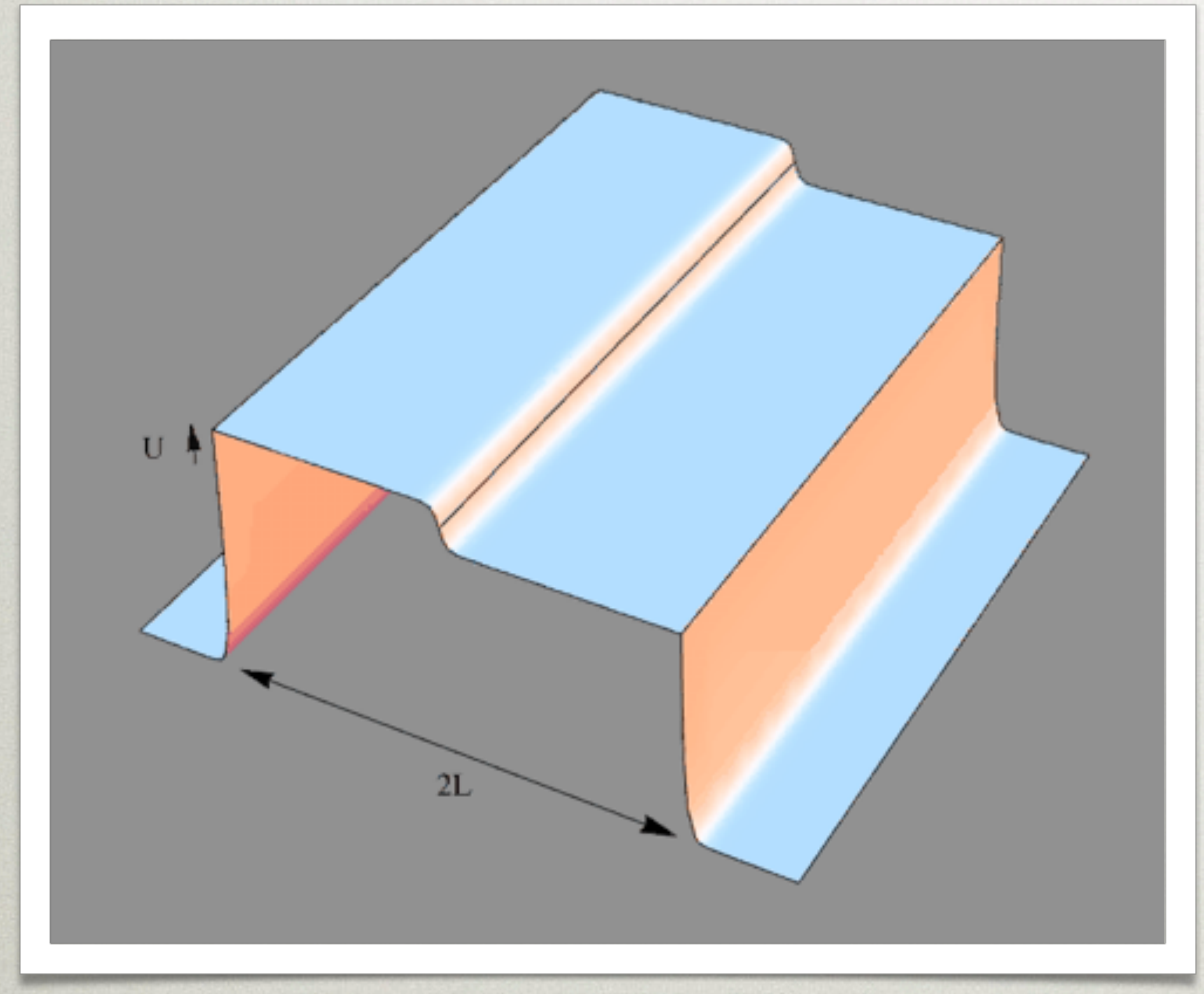
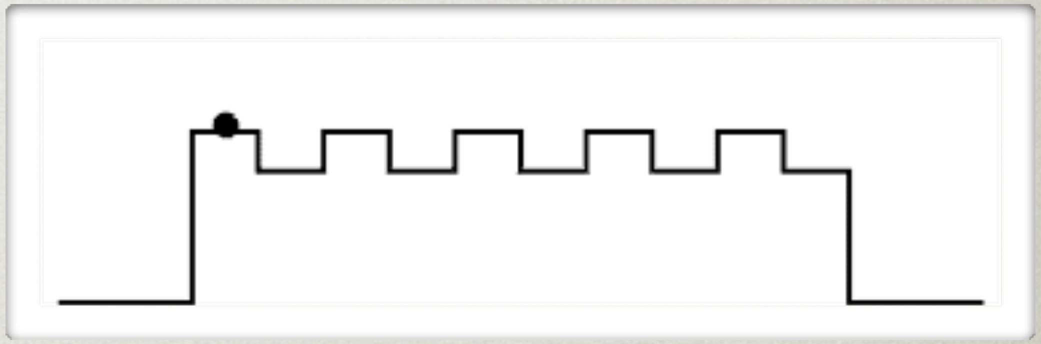


Quantum ratchet



Repeated ratchet units

S. Kohler, J. Lehmann and P. Hänggi, Physics Reports **406** 379 - 443 (2005)

Floquet theory

- **Floquet theory**: turns a periodic, time dependent problem into a static one
  - In the stationary limit, the propagators in the driven system obey

$$G(t,t') = G(t+T, t'+T)$$

  - One can write the time-averaged pumped current in terms of

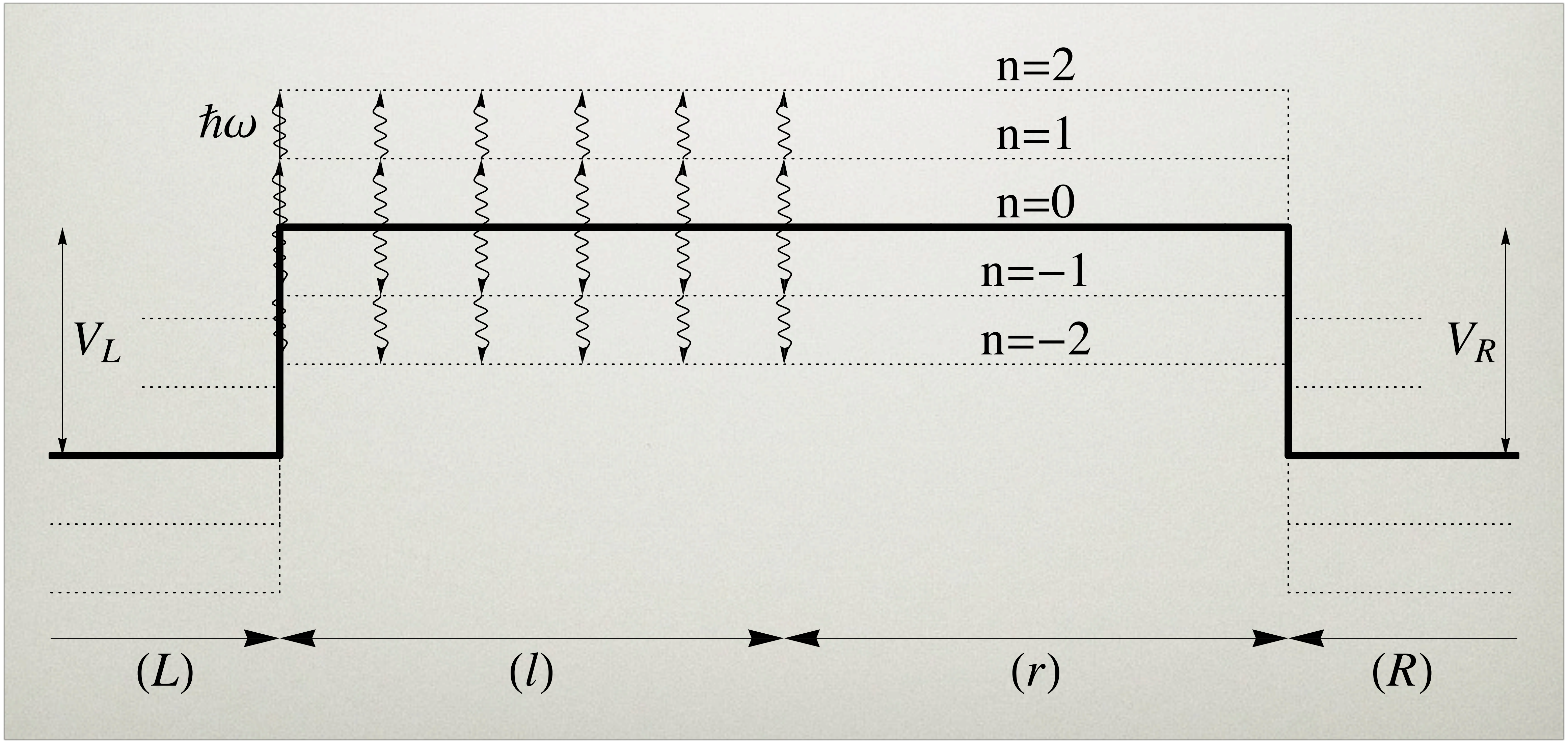
$$G^{(n)}(\epsilon) \equiv \frac{1}{T}\int_0^T d\bar{t}\, e^{in\omega\bar{t}} \int d\Delta T\, e^{i\epsilon\Delta t} G\left(\bar{t}+\frac{\Delta t}{2}, \bar{t}-\frac{\Delta t}{2}\right)$$

S. Kohler, J. Lehmann and P. Hänggi, Physics Reports **406** 379 - 443 (2005)

Floquet theory

- A propagator among coupled sidebands

$$H = \sum_n \left(H^{(0)} - n\hbar\omega \right) |n\rangle\langle n| + \frac{1}{2} \sum_n U(x) \left(|n+1\rangle\langle n| + |n\rangle\langle n+1| \right)$$



Floquet theory

- The time-averaged pumped current is:

$$\bar{I} = \frac{e}{h} \sum_{n=-\infty}^{\infty} \int d\epsilon \left[T^{(n)}_{L\to R}(\epsilon) - T^{(n)}_{R\to L}(\epsilon) \right] f(\epsilon)$$

where the Floquet transmissions are

$$T^{(n)}_{i\to j} = \Gamma_i(\epsilon + n\hbar\omega)\Gamma_j(\epsilon)|G^{(n)}_{i\to j}(\epsilon)|^2$$

Floquet theory

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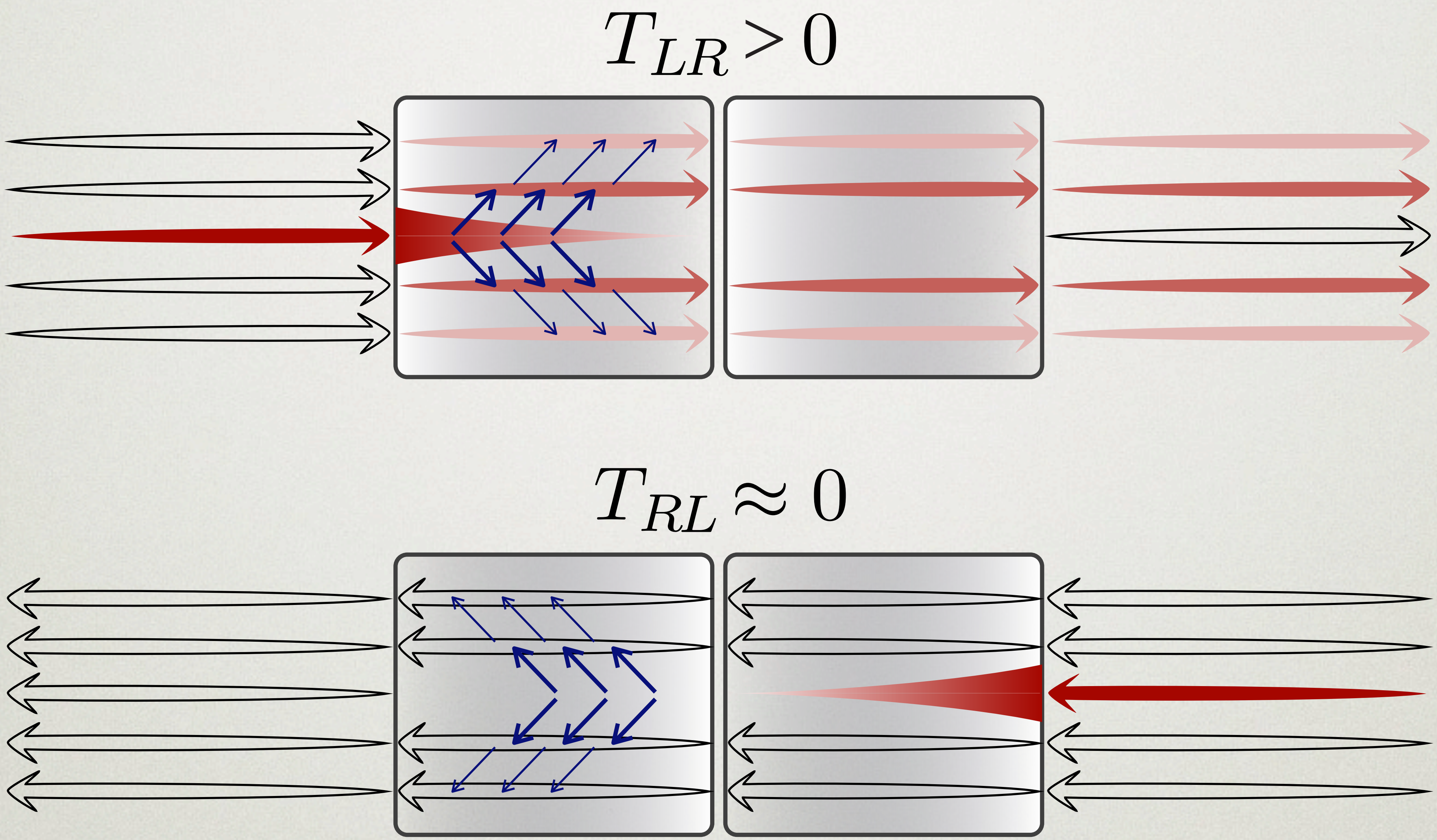
where the Floquet transmissions are

$$T^{(n)}_{i\rightarrow j} = \Gamma_i(\epsilon + n\hbar\omega)\Gamma_j(\epsilon)|G^{(n)}_{i\rightarrow j}(\epsilon)|^2$$

- At zero temperature

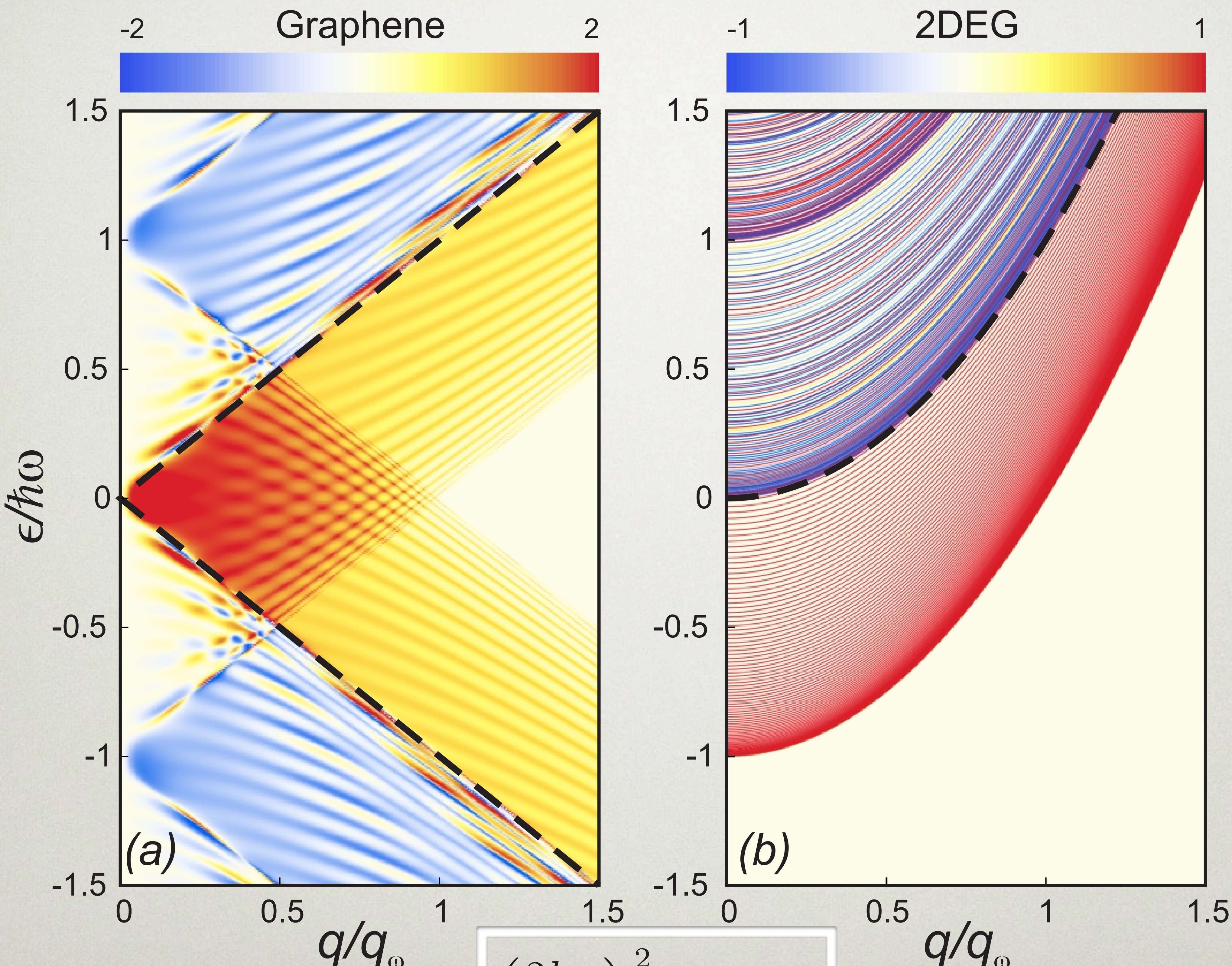
$$\frac{d\bar{I}}{dE_F} = \frac{e}{h} \sum_{n=-\infty}^{\infty} \left[T^{(n)}_{L\rightarrow R}(E_F) - T^{(n)}_{R\rightarrow L}(E_F) \right] = \frac{e}{h} \Delta T(E_F)$$

Transmission imbalance



$$\frac{d\bar{I}}{dE_F} = \frac{e}{h} \sum_{n=-\infty}^{\infty} \left[T^{(n)}_{L\to R}(E_F) - T^{(n)}_{R\to L}(E_F) \right] = \frac{e}{h} \Delta T(E_F)$$

Results

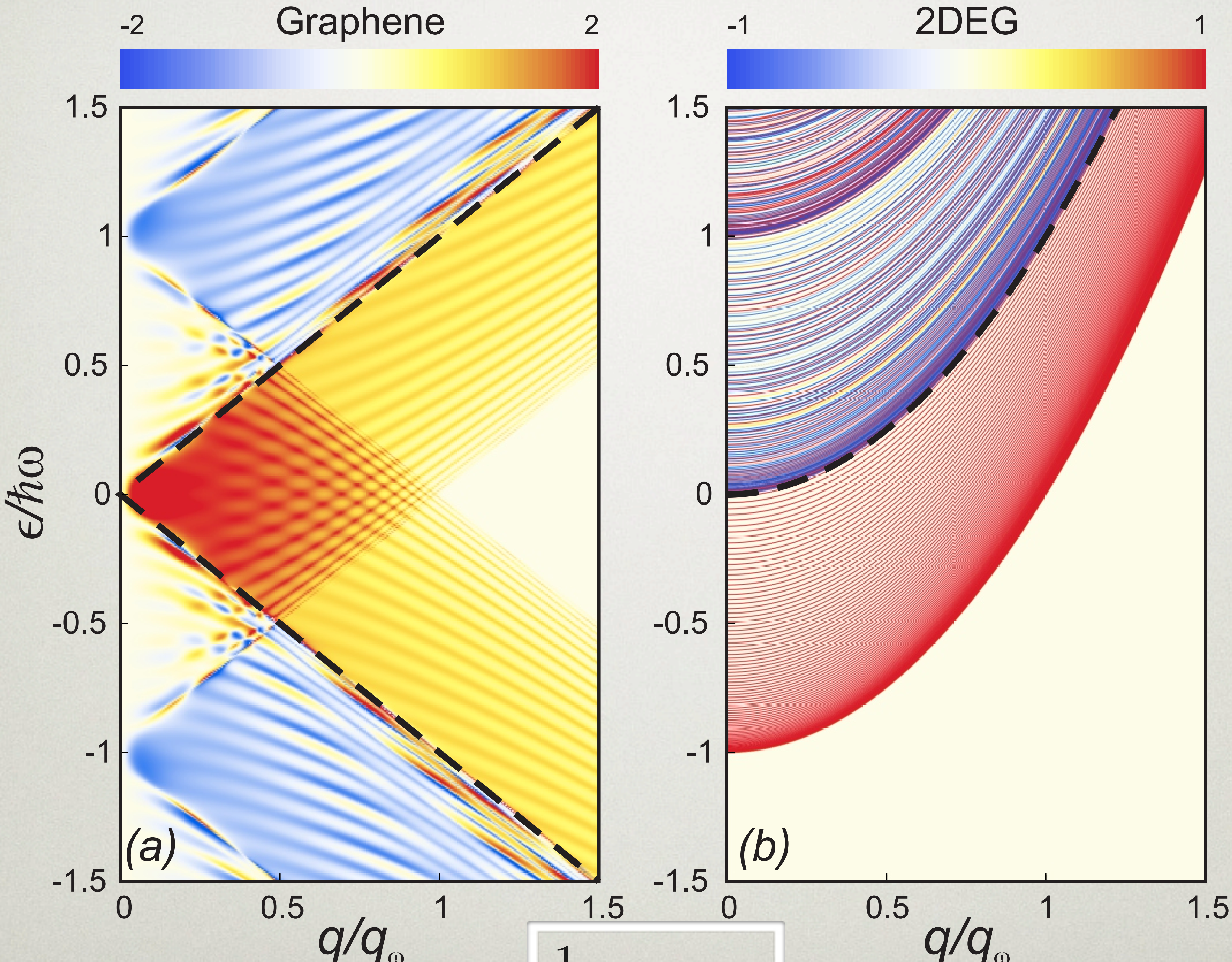


$$\left(\frac{2\hbar\omega}{U}\right)^2 \Delta T(q,\epsilon)$$

$q_\omega = \frac{\omega}{v_F}$

$q_\omega = \hbar\omega \frac{2m^* L}{\hbar^2}$

Results



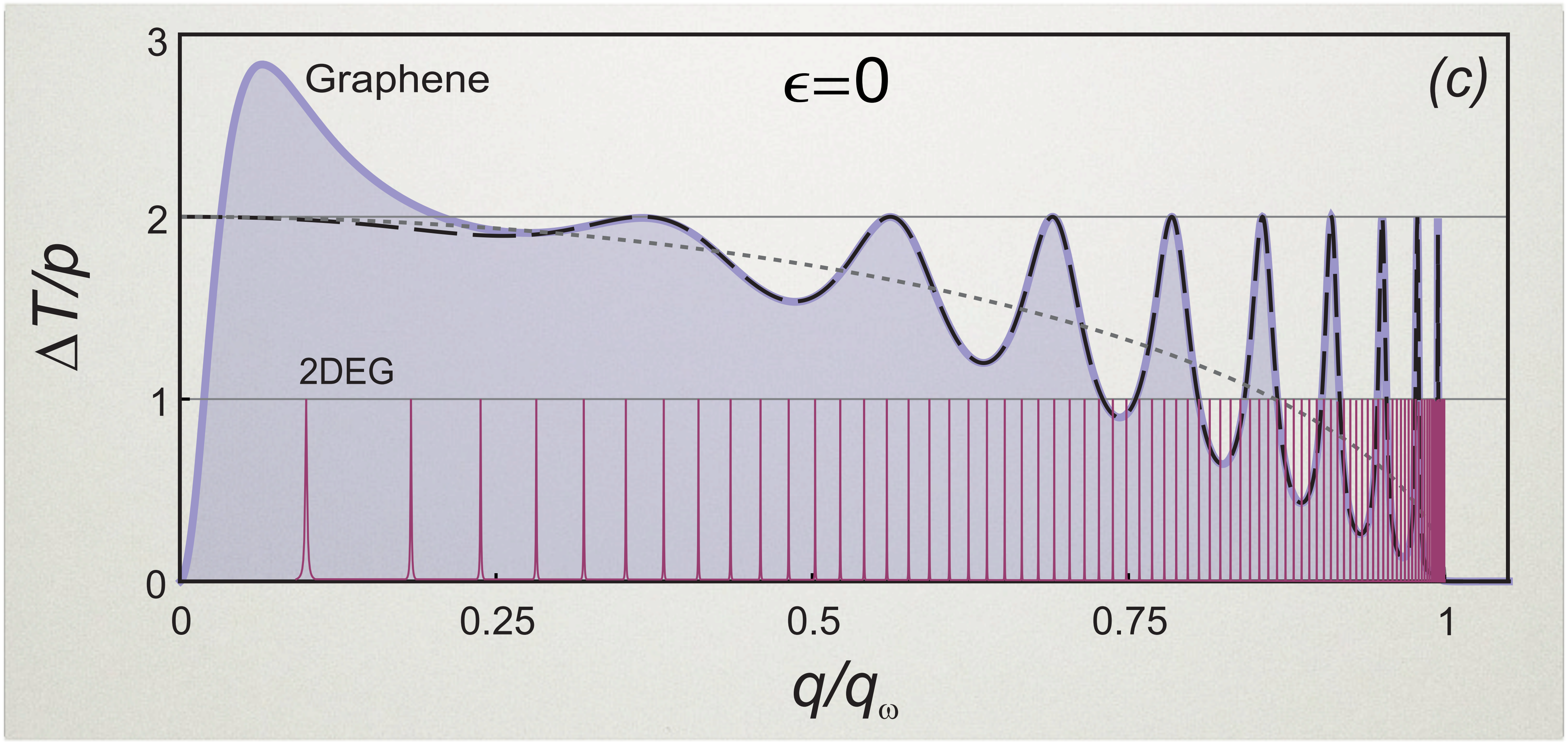
$$\frac{1}{p}\Delta T(q,\epsilon)$$

$$q_\omega = \frac{\omega}{v_F}$$

$$q_\omega = \hbar\omega\frac{2m^*L}{\hbar^2}$$

Results

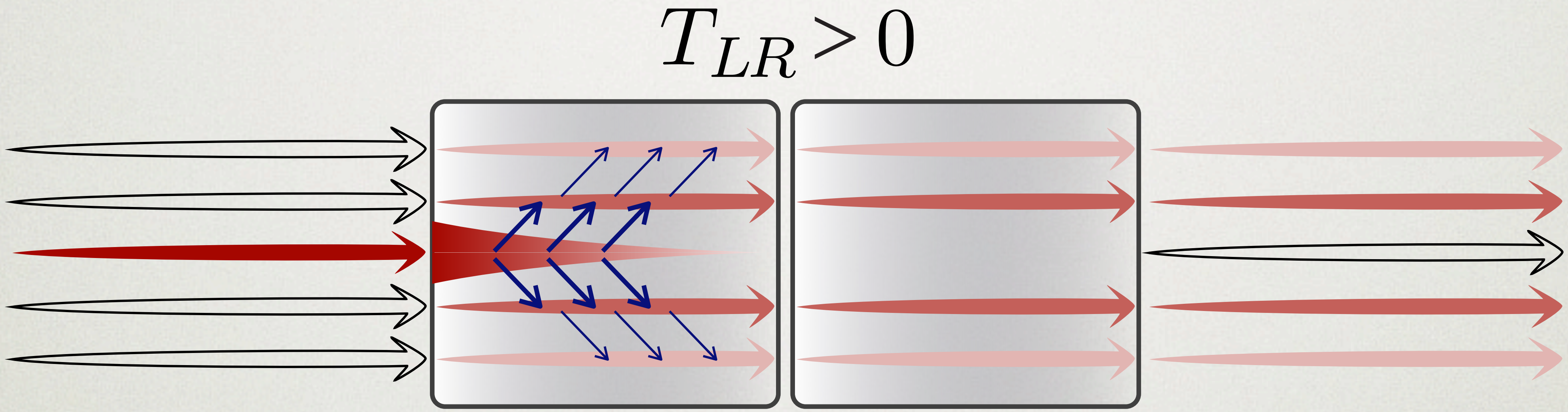
- Purely evanescent pumping response

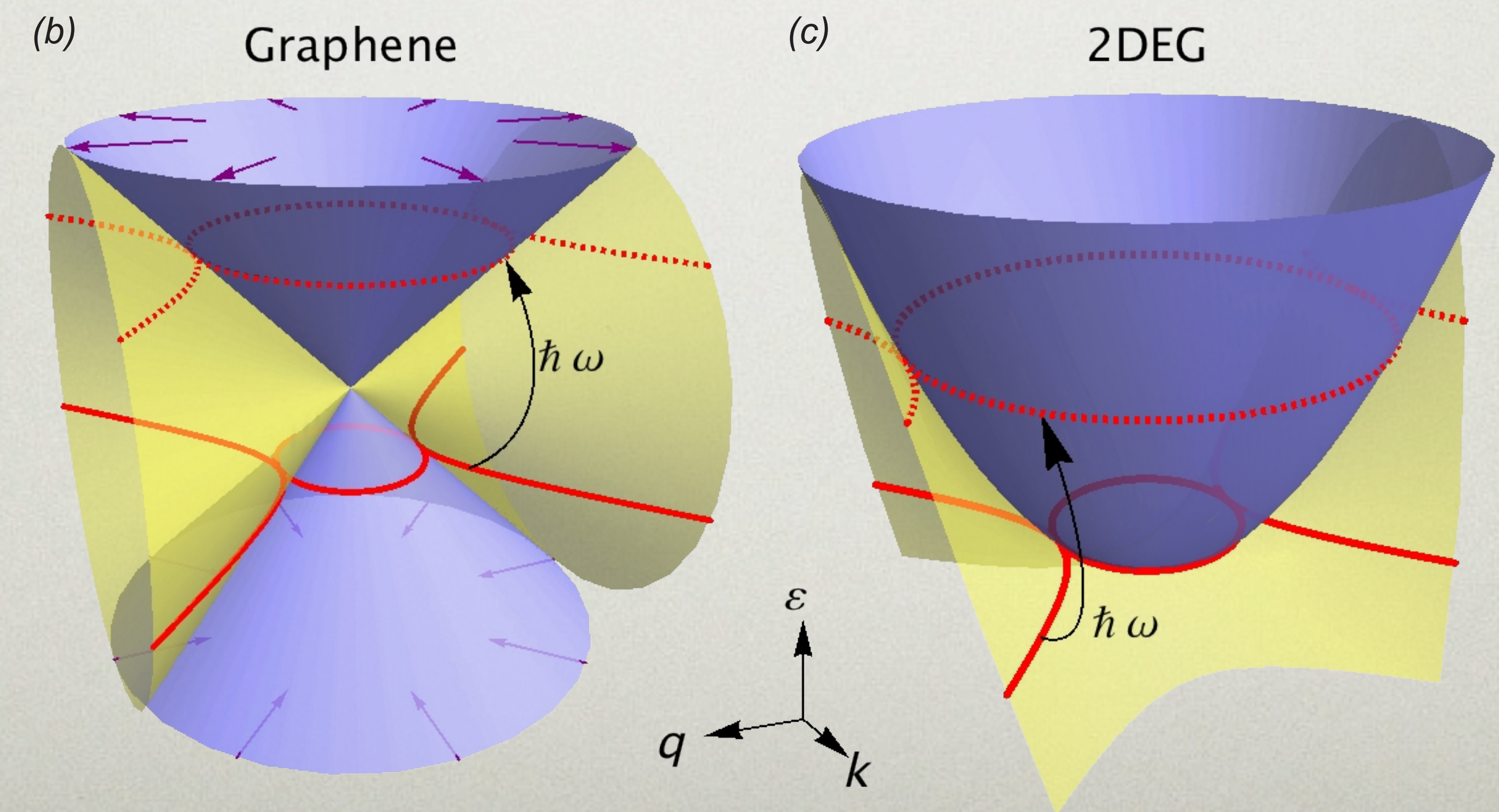


- Directed current! (always left-to-right)
- Relation to the static transmission?

Directed current

- Evanescent modes are pumped only in one direction





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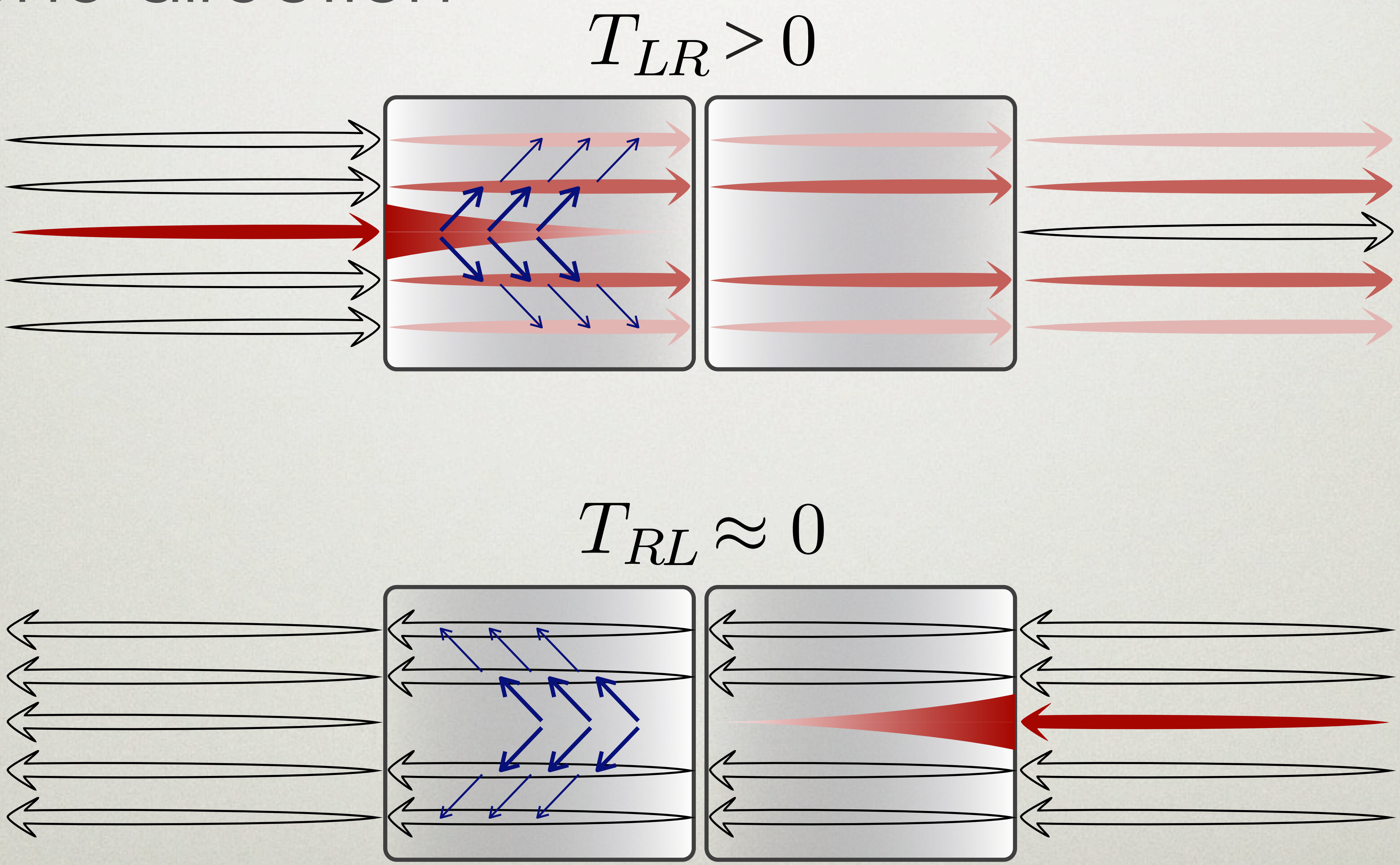
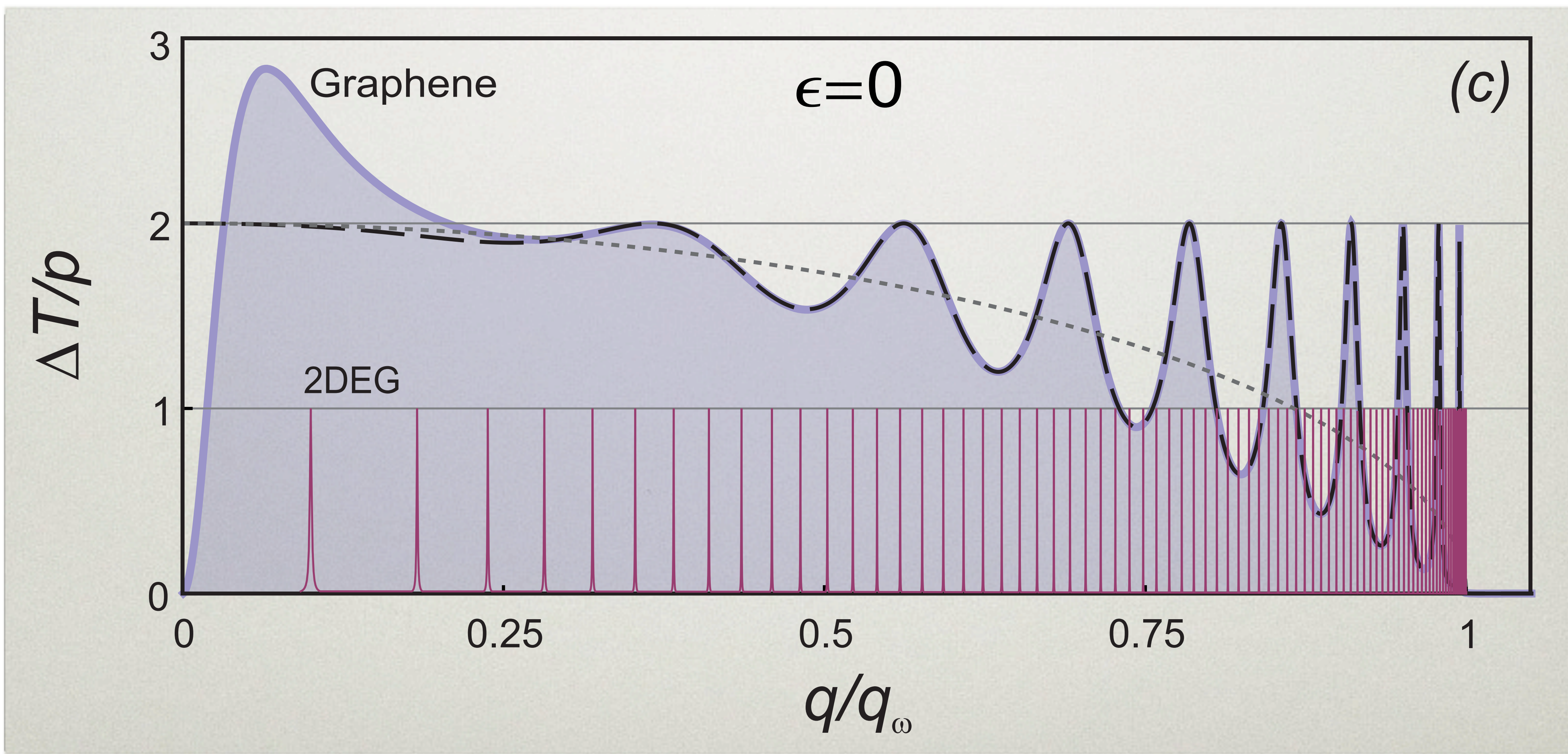


Pumping correspondence

- Pumping-transmission correspondence

$$\Delta T(\epsilon) = p\left[T(\epsilon + \hbar\omega) + T(\epsilon - \hbar\omega)\right] + \mathcal{O}\left(\left|e^{2ik_x L}\right|\right)$$

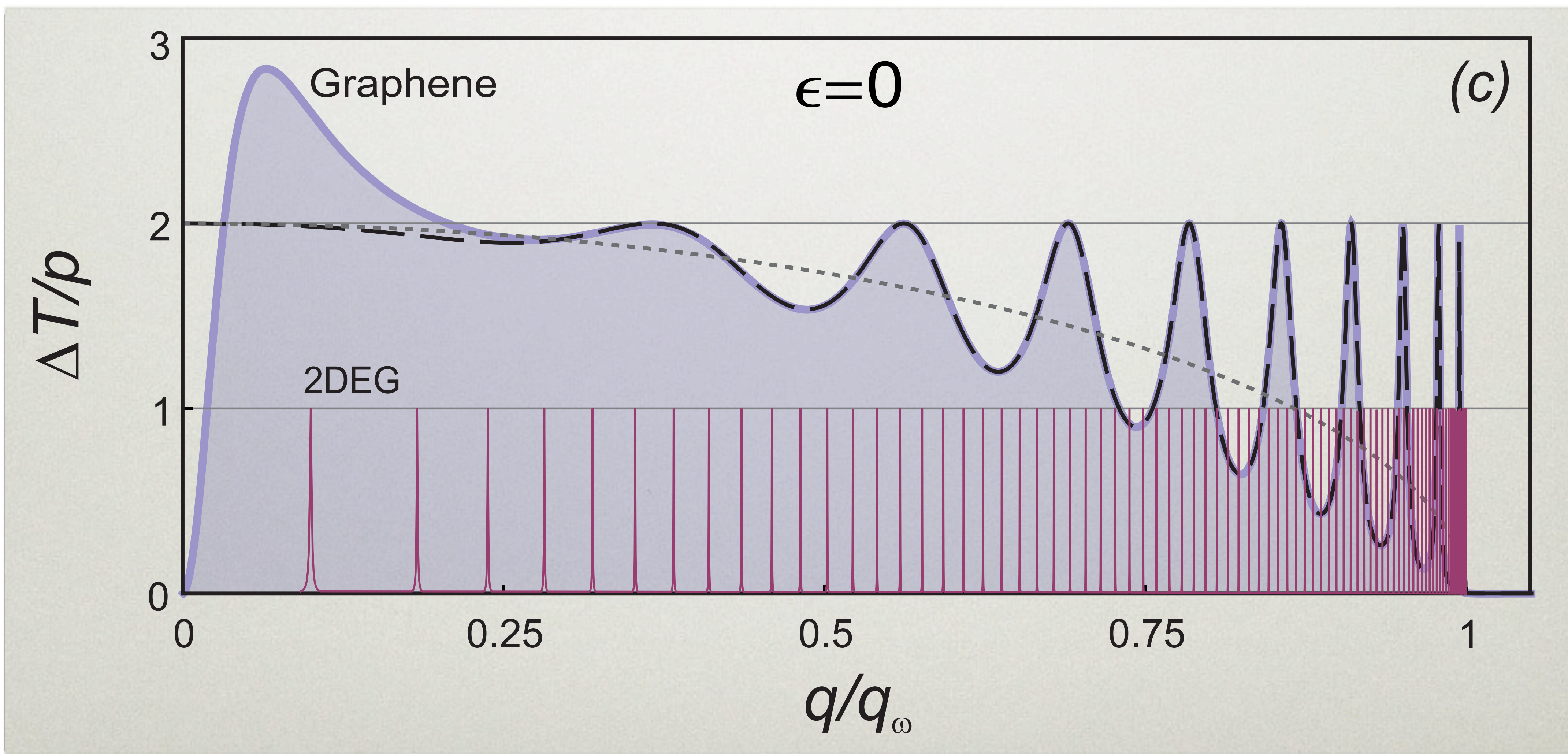


$p = U/2\hbar\omega$

Semiclassical limit

- Semiclassical limit in graphene (dotted)

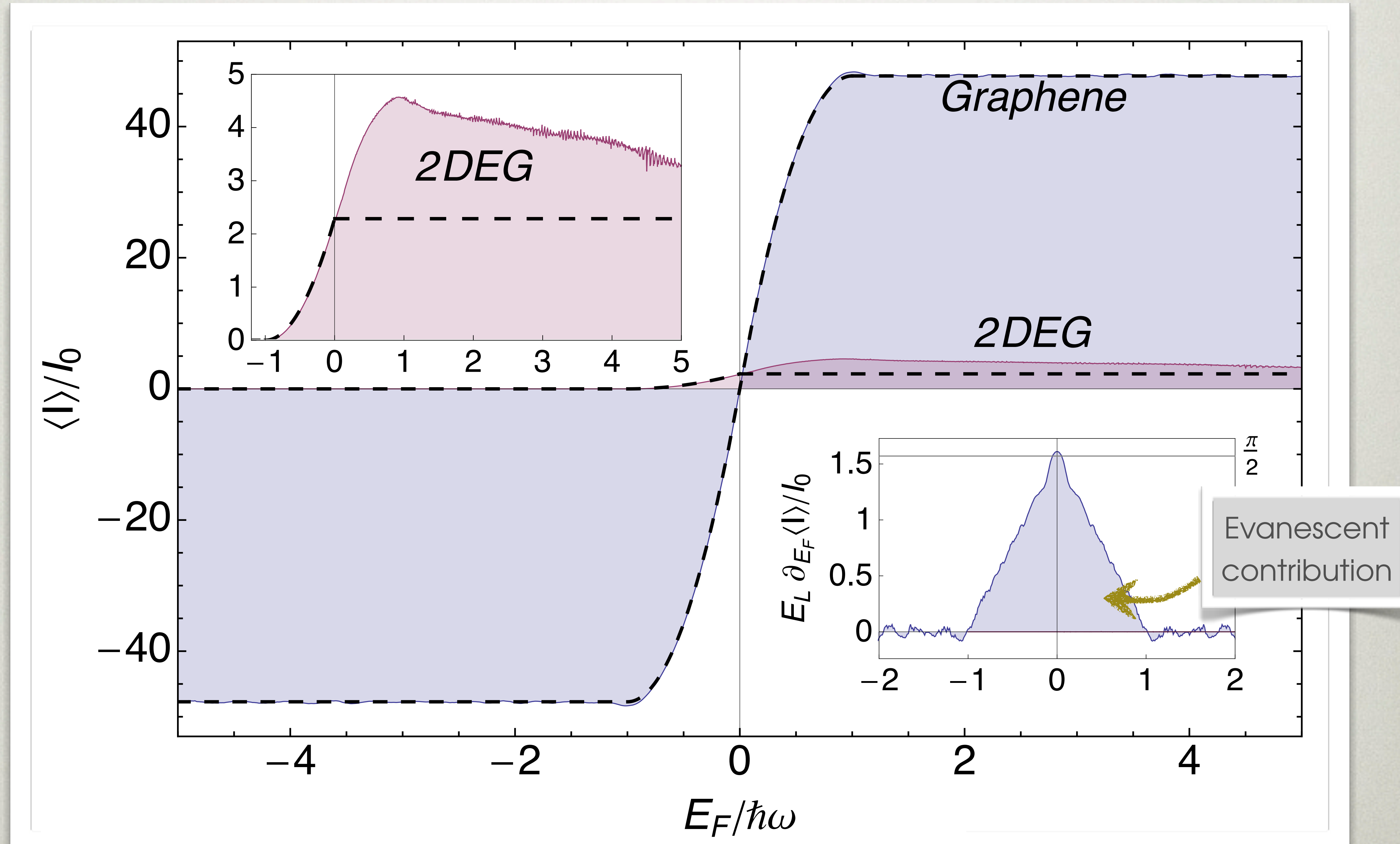
$$\Delta T(0) = 2p\sqrt{1 - \frac{v_F q}{\hbar\omega}} \qquad (\omega \gg E_L^{(G)} \sim 1\ \mathrm{THz})$$



$p = U/2\hbar\omega$

Total pumped current

- Chirality-enhanced evanescent pumping



$$I_0 = \frac{e}{2h}\frac{U^2}{\hbar\omega}\frac{W}{L}$$

Total pumped current

Total pumped current

- Weak driving limit: $\bar{I} = I_0\,\sigma\left(E_F/\hbar\omega\right)$

$$I_0 = \frac{e}{2h}\,\frac{U^2}{\hbar\omega}\,\frac{W}{L} \qquad \sigma(E_F) \equiv \frac{gL}{\hbar\omega}\int_{-\infty}^{E_F} d\epsilon \int_0^{\infty} dq\;\frac{\Delta T}{p}$$

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- Semiclassical approximation

$$\sigma_G = \frac{\pi\hbar\omega}{E_L^G} \times \begin{cases} (2 - |E_F|/\hbar\omega) E_F/\hbar\omega, & |E_F| < \hbar\omega \\ \pm 1, & |E_F| > \hbar\omega \end{cases}$$

$$\sigma_N \approx \frac{\pi\hbar\omega}{2 E_L^N k_F^{(\infty)} L} \times \begin{cases} 0, & E_F < -\hbar\omega \\ (1 + E_F/\hbar\omega)^2, & -\hbar\omega < E_F < 0 \; . \\ 1, & 0 < E_F \end{cases}$$

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- Graphene/2DEG efficiency ratio

$$\nu \equiv \frac{\langle I_G\rangle^{\max}}{\langle I_N\rangle^{\max}} = \frac{\sigma_G^{\max}}{\sigma_N^{\max}} = \frac{\hbar k_F^{(\infty)}}{m^* v_F} \approx 20.9$$

Conclusions

- Chirality opens W/L evanescent modes in graphene (minimal conductivity)
  - They also respond to *adiabatic* pumping
- Chirality opens all propagating modes
  - *Non-adiabatic* driving pumps any evanescent mode that can be excited to propagating.
  - Resulting current is directed (driven in the direction dictated by spatial asymmetry)