CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS
INSTITUTO DE CIENCIA DE MATERIALES DE MADRID (ICMM)

Centro de Física do Porto

Mobility of suspended monolayer and bilayer graphene at finite T

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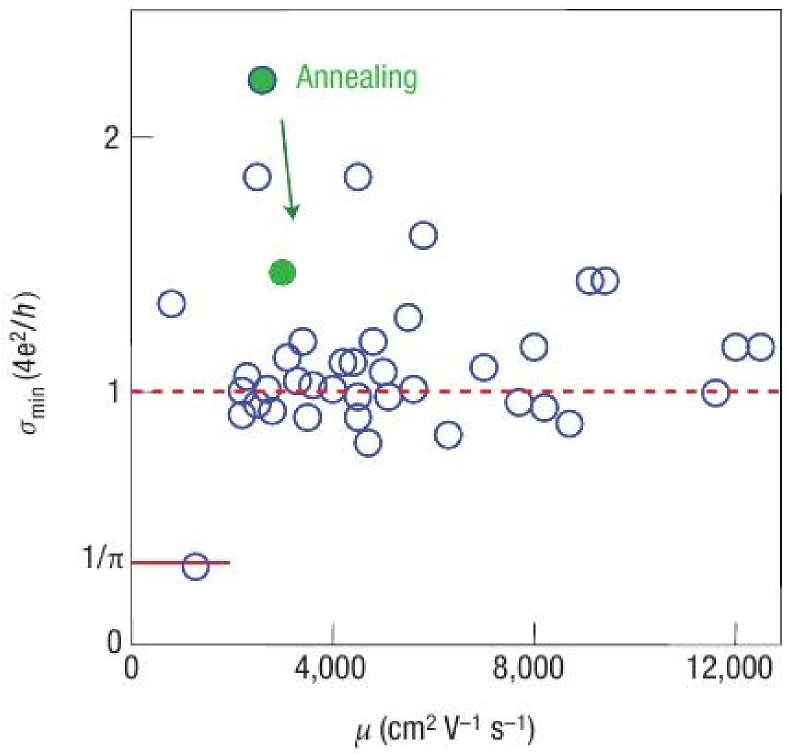
[4] University of Manchester, UK

*PRL **105**, 266601 (2010); arXiv:1008.2523 (Phys. E); arXiv:1102.0807*

ImagineNano – Graphene 2011, Bilbao, Spain *12/04/2011*

From high to ultra high mobility

- *Graphene's mobility is routinely high*
  - ~10^4 cm^2/Vs mobility at room T and high density

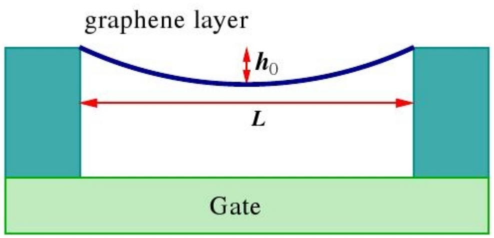


Geim and Novoselov, Nat. Mater. (2007)

- **Still limited by extrinsic scattering**

From high to ultra high mobility

- *Ultra high mobility in suspended samples at low T*
  - $>10^5$ cm^2/Vs mobility after current annealing



Solid State Commun. (2008)

Ultrahigh electron mobility in suspended graphene

K.I. Bolotin[a,*], K.J. Sikes[b], Z. Jiang[a,d], M. Klima[c], G. Fudenberg[a], J. Hone[c], P. Kim[a], H.L. Stormer[a,b,e]

[a] *Department of Physics, Columbia University, New York, NY 10027, USA*

Nature Nanotech. (2008)

Approaching ballistic transport in suspended graphene

XU DU, IVAN SKACHKO, ANTHONY BARKER AND EVA Y. ANDREI*

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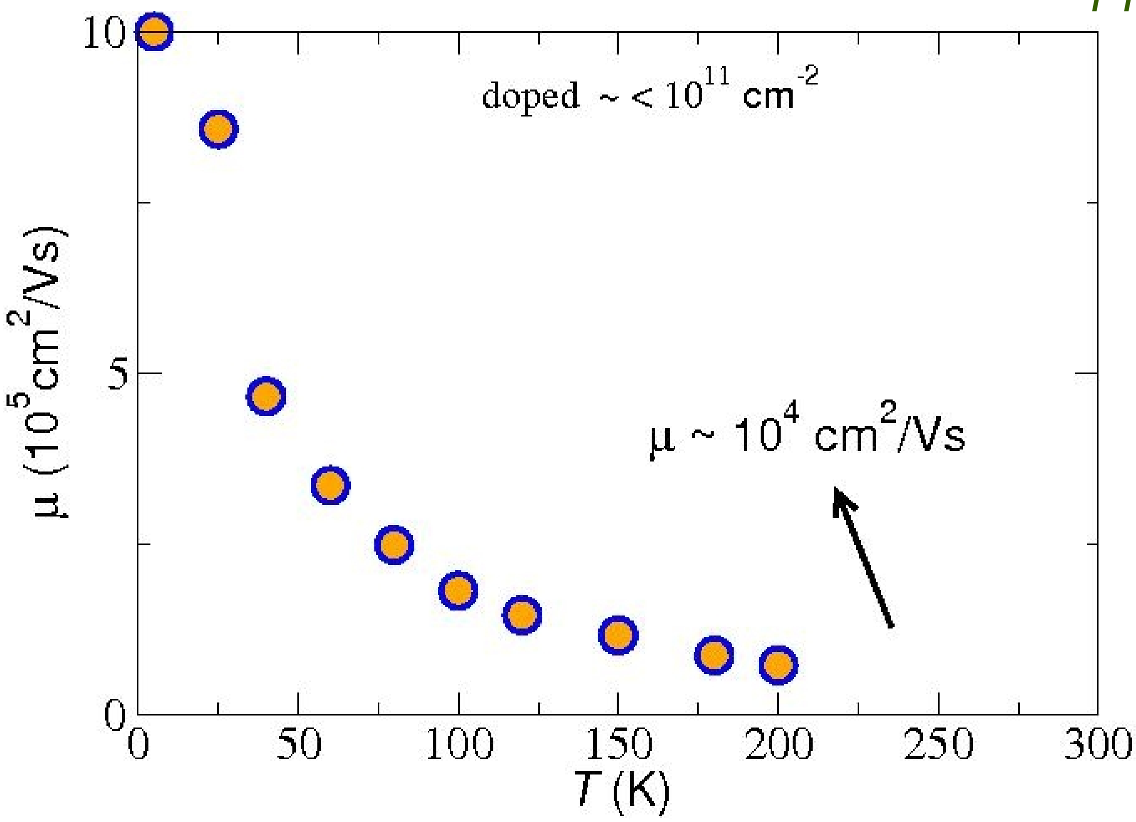
*e-mail: eandrei@physics.rutgers.edu

From high to ultra high mobility

- *Ultra high mobility in suspended samples at low T*
  - ~10^6 cm^2/Vs mobility (Manchester group)

*PRL **105**, 266601 (2010)*



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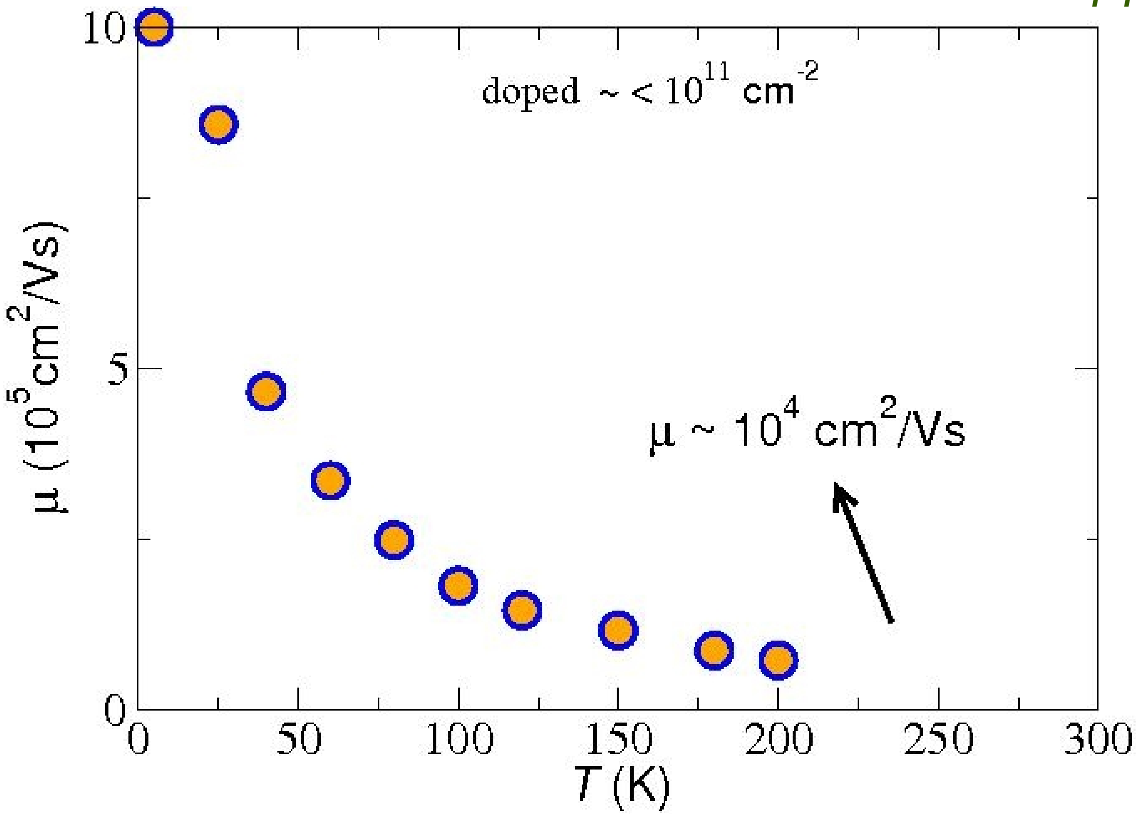


Not explained by acoustic in-plane phonons!

~10^5 cm^2/Vs at room T Hwang & Das Sarma, PRB (2008)

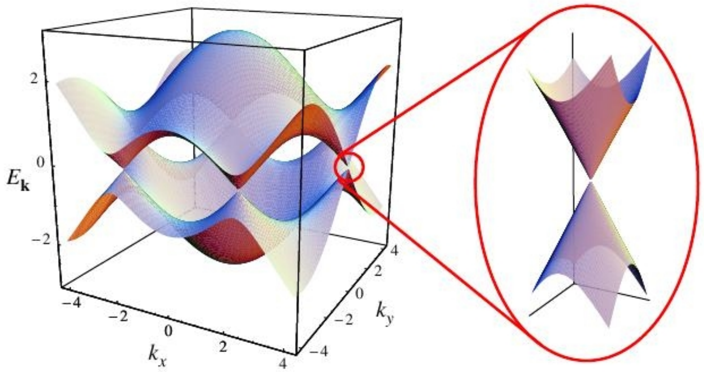
Ingredients monolayer

- *The metallic side*

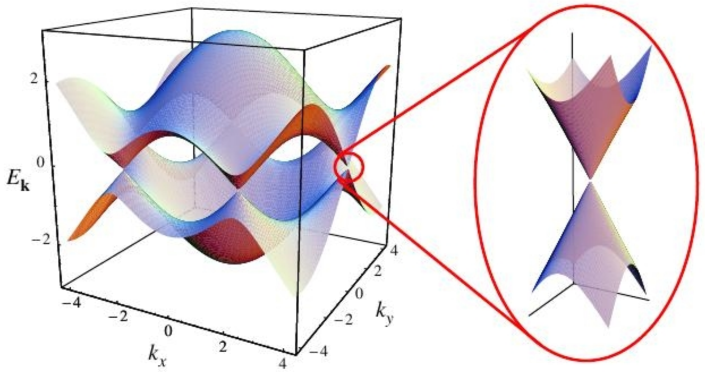
$$\mathcal{H}_0 \simeq \frac{\hbar}{i} v_F \left[\begin{array}{cc} 0 & (\partial_x - i\partial_y) \\ (\partial_x + i\partial_y) & 0 \end{array} \right]$$

Wallace,

Semenoff, 1984



Ingredients monolayer



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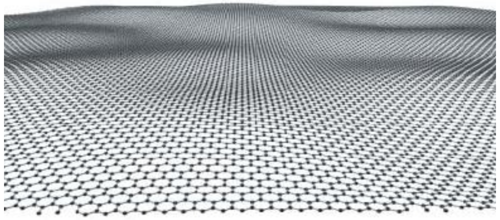
$$\mathcal{H}_0 \simeq \frac{\hbar}{i} v_F \begin{bmatrix} 0 & (\partial_x - i\partial_y) \\ (\partial_x + i\partial_y) & 0 \end{bmatrix}$$

Wallace,

Semenoff, 1984

- *The elastic side*



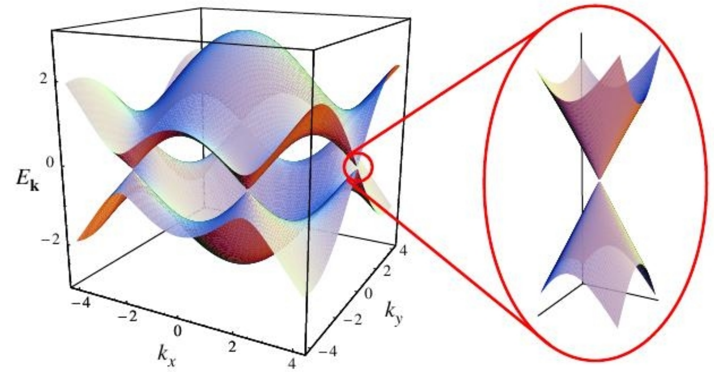
$$\mathcal{F} = \frac{1}{2}\int dxdy(\lambda u_{ii}^2 + 2\mu u_{ij}^2) + \frac{1}{2}\kappa\int dxdy(\nabla^2 h)^2$$

stretching

bending

Landau, Lifshitz, *Elasticity*
Nelsson, Piran, Weinberg

Ingredients monolayer



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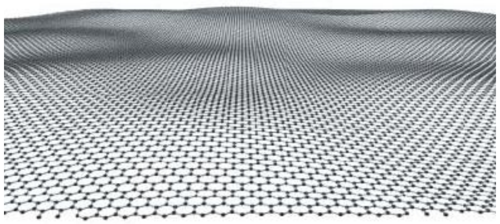


$$\mathcal{F} = \underbrace{\frac{1}{2}\int dxdy(\lambda u_{ii}^2 + 2\mu u_{ij}^2)}_{\text{stretching}} + \underbrace{\frac{1}{2}\kappa \int dxdy(\nabla^2 h)^2}_{\text{bending}}$$

Landau, Lifshitz, *Elasticity*
Nelsson, Piran, Weinberg

- *The coupling*

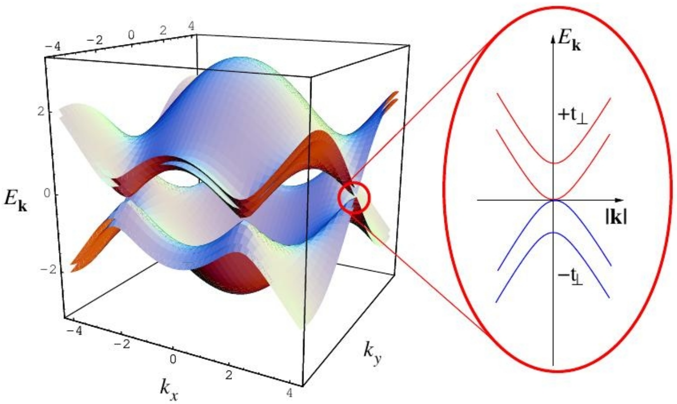
Castro Neto *et. al.*, RPM (2009); Vozmediano *et. al.*, Phys. Rep. (2010)

$$\mathcal{H}_{ep} = \mathbf{1}V + v_F e(A_x \mp iA_y)|_{off-diag}$$

$$\boxed{V = g_0 u_{ii}}$$ **+ screening**

$$\boxed{e\mathbf{A} = \frac{\hbar\beta}{a}\left[\ \tfrac{1}{2}(u_{xx} - u_{yy}), \quad -u_{xy}\ \right]}$$

Ingredients bilayer



- *The metallic side*

$$\mathcal{H}_0 \simeq \frac{\hbar^2}{2m}\begin{bmatrix} 0 & (\partial_x - i\partial_y)^2 \\ (\partial_x + i\partial_y)^2 & 0 \end{bmatrix}$$

McCann and Falko, PRL (2006)

- *The elastic side*

$$\mathcal{F} = \frac{1}{2}\int dxdy(\lambda u_{ii}^2 + 2\mu u_{ij}^2) + \frac{1}{2}\kappa \int dxdy(\nabla^2 h)^2$$

stretching

bending

Zakharchenko *et al.*, PRB (2010)

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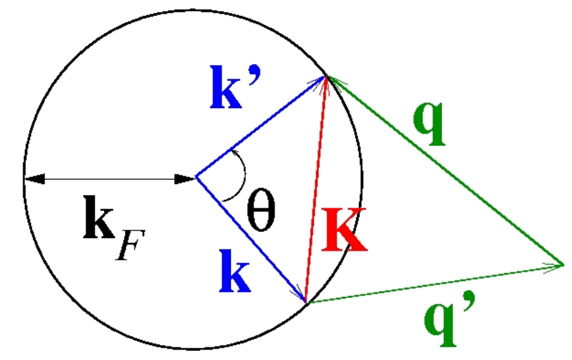
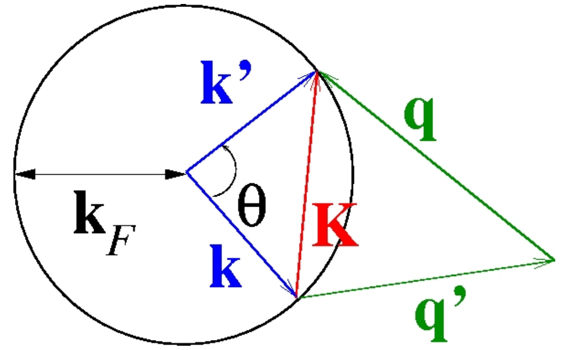
Resistivity – flexural phonons

- *Assumptions*
  - Harmonic approximation
  - Doped regime:

    semi-classical transport + Born approx.
  - Quasi-elastic approximation
  - High T ($T \gg T_{BG}$)



Resistivity – flexural phonons



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Mariani & von Oppen, PRB (2010)

PRL ***105****, 266601 (2010)*

Monolayer

$$\varrho \approx \left(\frac{g^2}{2} + \frac{\hbar^2 v_F^2 \beta^2}{4a^2}\right) \frac{(k_B T)^2}{64 \hbar e^2 \kappa^2 v_F^2 k_F^2} \ln\left(\frac{k_B T}{\hbar \omega_c}\right)$$

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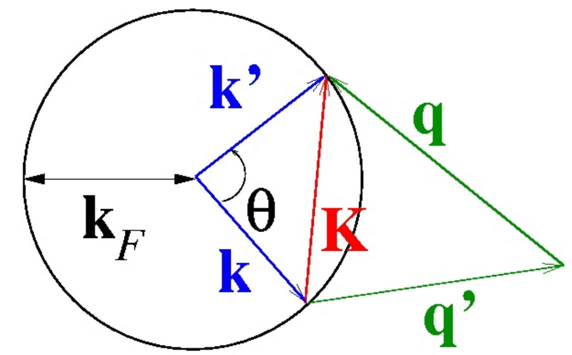

Mariani & von Oppen, PRB (2010)

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Bilayer

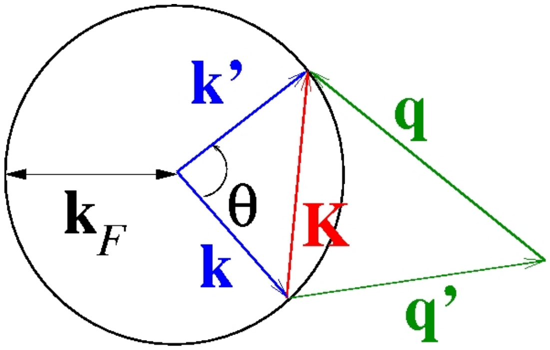
$$\varrho \approx \left(g^2 + \frac{\hbar^2 v_F^2 \beta^2}{8a^2}\right) \frac{(k_B T)^2}{64\hbar e^2 \kappa^2 v_F^2 k_F^2} \ln\left(\frac{k_B T}{\hbar\omega_c}\right)$$

arXiv: 1102.0807

Mobility - Monolayer

- *Mobility* $1/\mu = \varrho e n$

$$\boxed{\mu \approx \frac{64\pi\kappa^2}{\frac{g^2}{2} + \frac{\hbar^2 v_F^2 \beta^2}{4a^2}} \frac{\hbar e v_F^2}{(k_B T)^2} \ln\left(\frac{k_B T}{\hbar\omega_c}\right)}$$



PRL ***105****, 266601 (2010)*

$\kappa \approx 1\ \mathrm{eV}$ Zakharckenko *et al*., PRL (2009)

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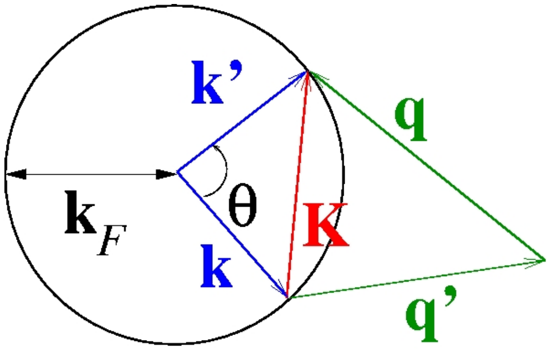

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$$\mu_{300\mathrm{K}} \sim 1 \text{ m}^2/\text{Vs}$$

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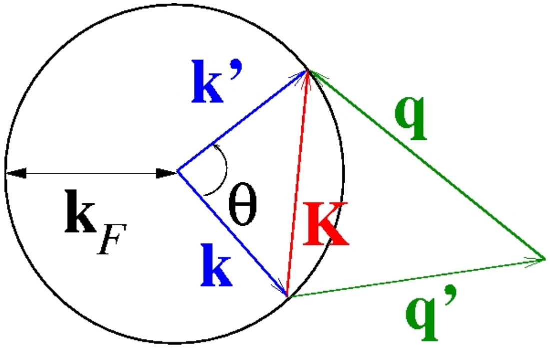

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- **Logrithmic divergence**

For flexural phonons $\mathcal{N}_{\mathbf{q}} \approx \int d\mathbf{q}\, n_{\mathbf{q}} \approx \int d\omega \mathcal{D}(\omega) n(\omega)$ logarithmically diverges in the infrared $(q \to 0)$

$$\boxed{\omega_{\mathbf{q}}^F = \alpha q^2}$$

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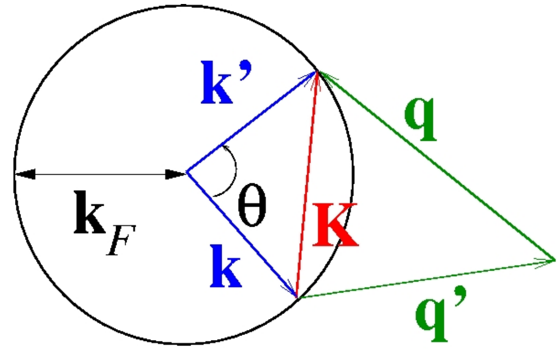

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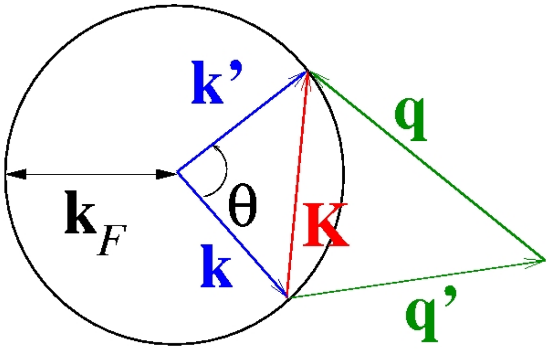
Onset of anharmonic effects

natural *ir* cutoff at long wave lengths $\hbar\omega_c \ll k_B T$

Zakharchenko *et. al.*, PRB (2010)

Mobility – Bilayer

- *Mobility* $1/\mu = \varrho e n$



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arXiv: 1102.0807

$\boxed{\kappa \approx 2 \text{ eV}}$ Zakharckenko *et al.*, PRB (2010)

$g \sim 3 \text{ eV}$ Choi *et. al.*, PRBr (2010)

$\beta = -\partial \log t / \partial \log a \sim 2-3$ Castro Neto *et. al.*, RPM (2009); Vozmediano *et. al.*, Phys. Rep. (2010)

$$\boxed{\mu_{300\text{K}} \sim 10 \text{ m}^2/\text{Vs}}$$

- **Logrithmic divergence**

For flexural phonons $\mathcal{N}_{\mathbf{q}} \approx \int d\mathbf{q}\, n_{\mathbf{q}} \approx \int d\omega \mathcal{D}(\omega) n(\omega)$ logarithmically diverges in the infrared $(q \to 0)$

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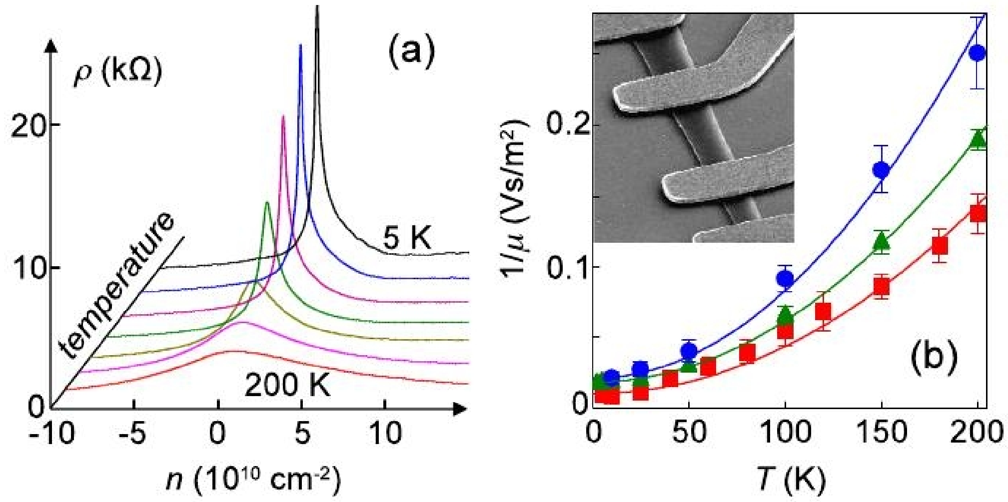
Resistivity / mobility in experiments



PRL ***105****, 266601 (2010)*

- Experiment: $1/\mu \approx 1/\mu(T \to 0) + \gamma T^2$
  - $\gamma \sim 3 - 6 \times 10^{-6}\ \mathrm{Vs}/(\mathrm{mK})^2$

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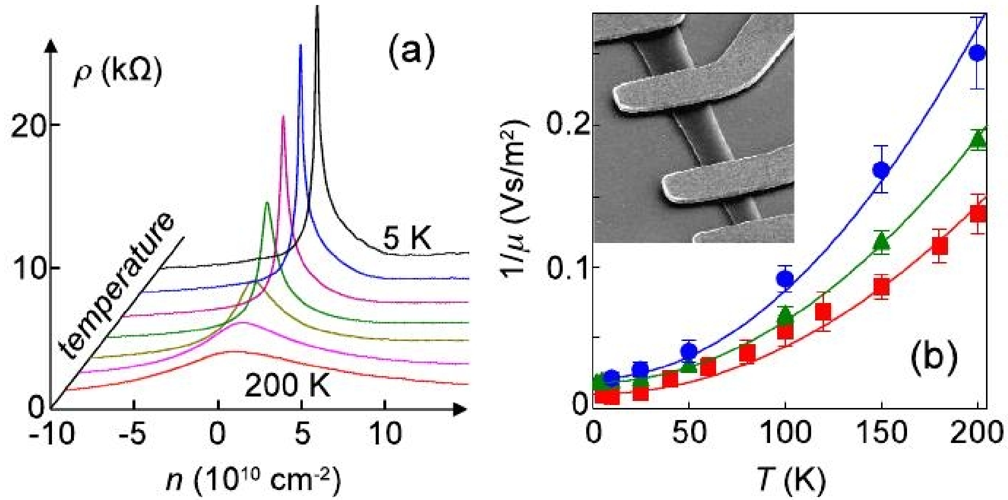


*PRL **105**, 266601 (2010)*

- Experiment: $1/\mu \approx 1/\mu(T \to 0) + \gamma T^2$
  - $\gamma \sim 3 - 6 \times 10^{-6}$ Vs/(mK)^2
- Theory (gives T^2):

$$\frac{1}{\mu} \approx \left(\frac{g^2}{2} + \frac{\hbar^2 v_F^2 \beta^2}{4a^2} \right) \frac{(k_B T)^2}{64\pi\hbar e \kappa^2 v_F^2} \ln \left(\frac{k_B T}{\hbar \omega_c} \right)$$

  - $\gamma \approx 3 \times 10^{-6}$ Vs/(mK)^2 (without adjustable parameters, ignoring logarithm)
  - different cutoff in $\ln(T/\omega_c)$ provides difference between samples

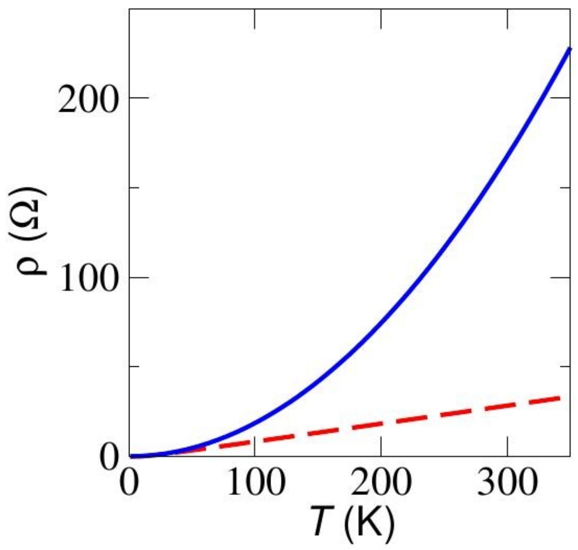
ω_c **depends on strain** Roldán *et. al.*, arXiv:1101.6026

~ 2 **factor compatible with u~10^{-4}** Chen *et. al.*, Nature Nanotech. (2009)

Conclusions

- *Room T resistivity dominated by flexural phonon contribution*

$$\boxed{\mu_{300\mathrm{K}} \sim 1\ \mathrm{m}^2/\mathrm{Vs}}$$



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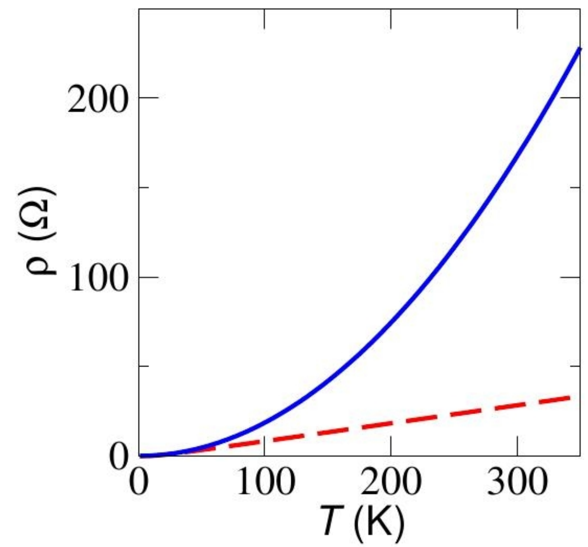
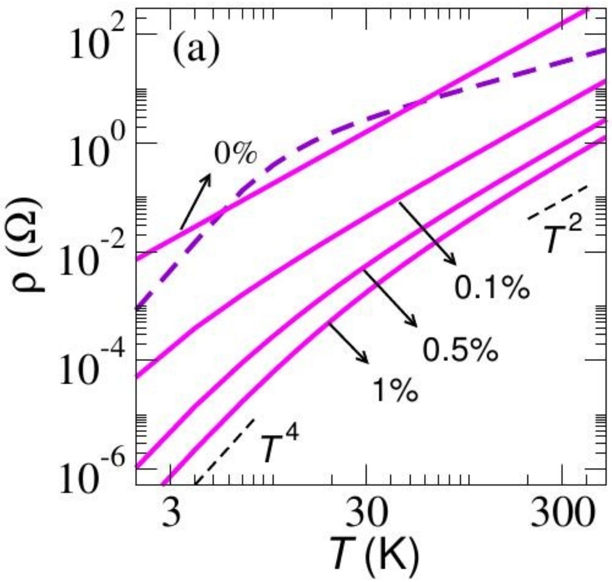


- *How to avoid flexural phonons?*
  - Strain strongly suppresses scattering by flexural phonons
  - Go back to substrate (ex: **BN**)



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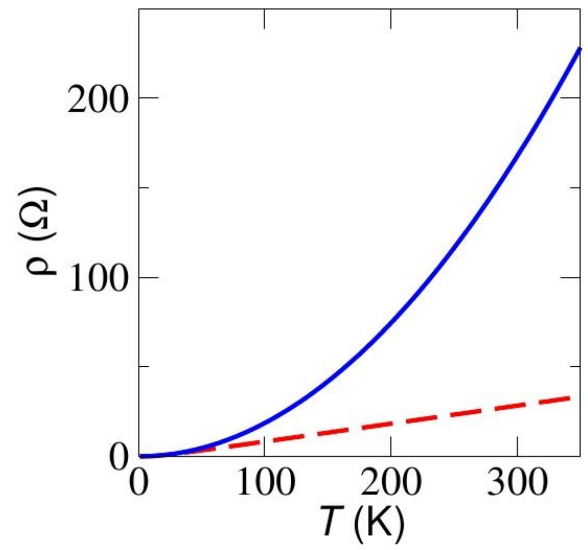

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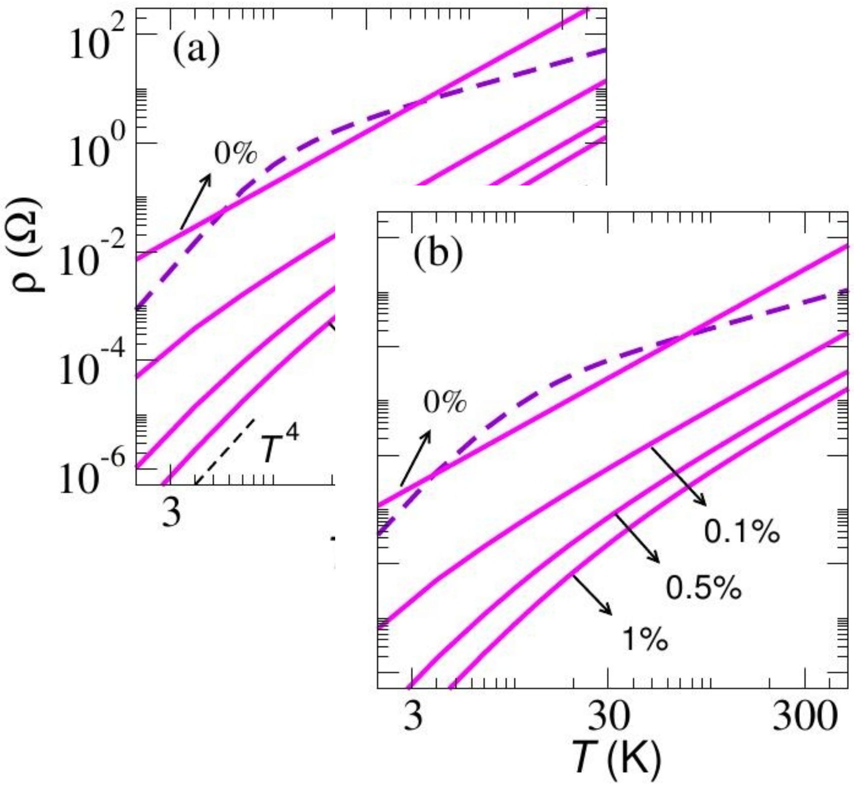
- *Bilayer graphene*
  - Qualitatively similar
  - Quantitatively different



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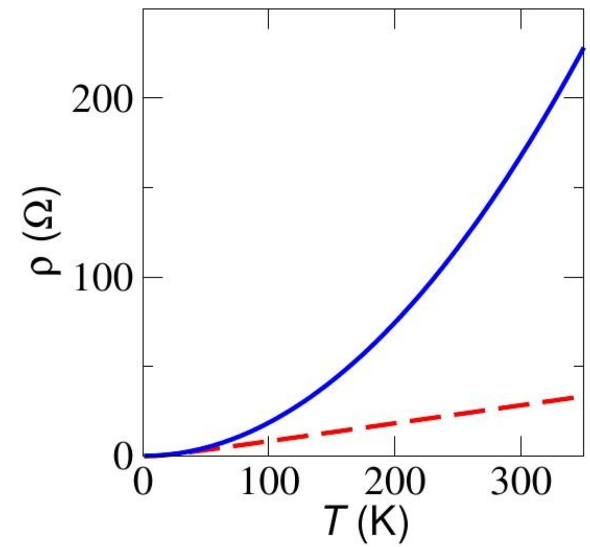

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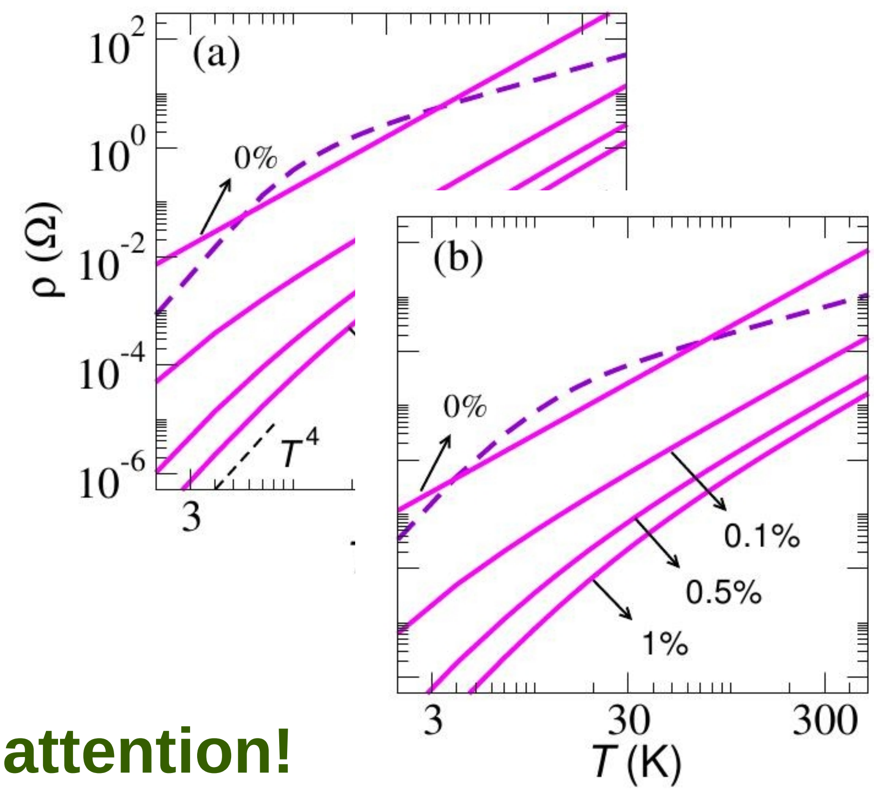


- *Bilayer graphene*
  - Qualitatively similar
  - Quantitatively different

Thank you for your attention!

Appendix

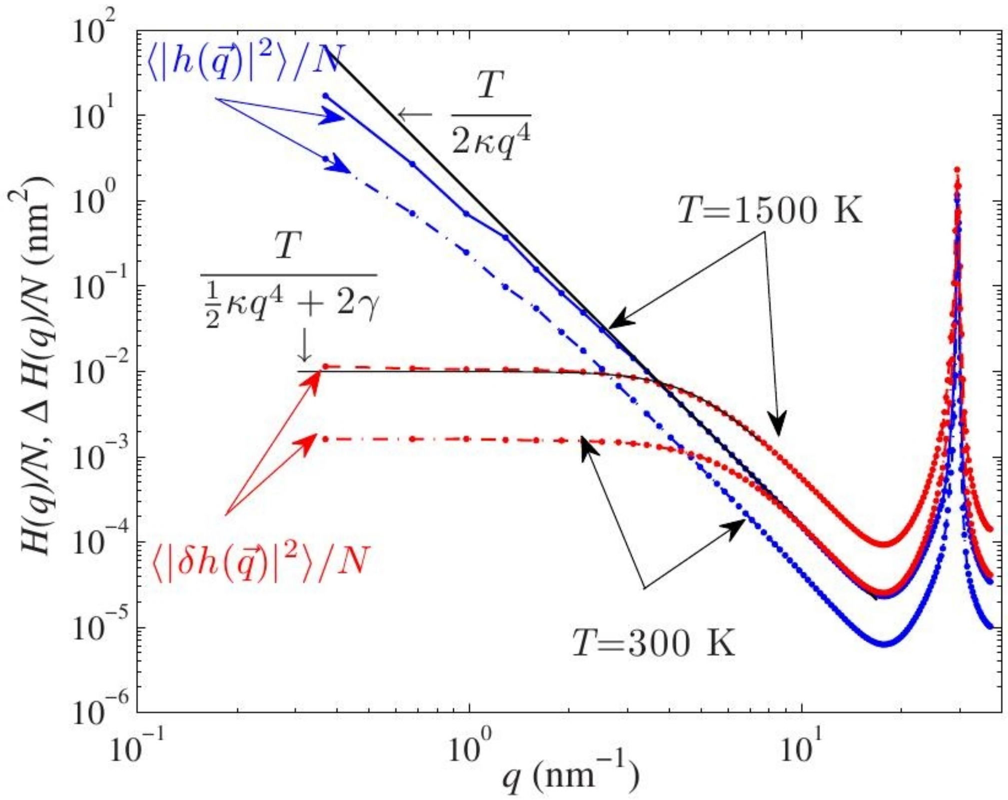
Just a thicker membrane

- *Reasonable approach for acoustic phonons*

PHYSICAL REVIEW B **81**, 235439 (2010)

Atomistic simulations of structural and thermodynamic properties of bilayer graphene

K. V. Zakharchenko, J. H. Los, M. I. Katsnelson, and A. Fasolino



Zakharchenko *et al.*, PRB (2010)

Electron–phonon coupling

- *Two types of e-ph terms allowed by symmetry*

$$\mathcal{H}_{ep} = \mathbf{1}V + v_F e \boldsymbol{\sigma}\cdot\mathbf{A}$$

Scalar potential like — Vector potential like

- **Deformation potential**

$$\boxed{V = g_0 u_{ii}}$$

Which *g* ???

$$g_0 \sim 20-30 \text{ eV}$$ Suzuura & Ando, PRB (2002)

$$g \sim 3 \text{ eV}$$ Choi *et. al.*, PRBr (2010)

$$g_{exp} \sim 17\pm 1 \text{ eV}$$ Chen *et. al.*, Nat. Nano. (2008)

Include screening!

$$V_{\mathbf{k},\mathbf{k+q}} = \frac{g_0}{\epsilon(\mathbf{q})}u_{ii}(\mathbf{q}) \rightarrow \frac{g_0}{\epsilon_{TF}(k_F)} \equiv g \sim 3\,\text{eV}$$

- **Bond length change**

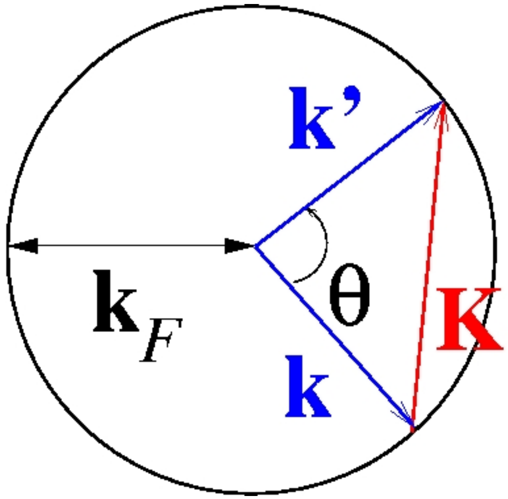
$$\boxed{e\mathbf{A} = \frac{\hbar\beta}{a}\left[\ \tfrac{1}{2}(u_{xx}-u_{yy}), \quad -u_{xy}\ \right]}$$

$$\beta = -\partial\log t/\partial\log a \sim 2-3$$

$$v_F\hbar\beta/a \approx 10-15\,\text{eV}$$

Castro Neto *et. al.*, RPM (2009); Vozmediano *et. al.*, Phys. Rep. (2010)

Resistivity – in-plane phonons



- *Assumptions*
  - Doped regime:

    semi-classical transport + Born approx.
  - Quasi-elastic approximation
- *High T ($T >> T_{BG}$)*

$$\varrho_{in} \approx \left(2g^2 + \frac{\hbar^2 v_F^2 \beta^2}{2a^2}\frac{v_L^2}{\bar{v}^2}\right)\frac{\pi k_B T}{4\hbar\rho e^2 v_L^2 v_F^2}$$

$g_{exp} \sim 17 \pm 1\,\mathrm{eV}$

$g \sim 3$ eV

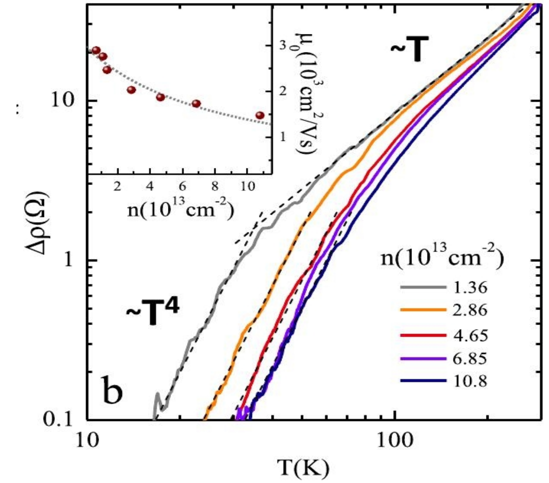
$\beta \sim 2-3$

Chen *et. al.*, Nat. Nano. (2008)

- *Low T ($T << T_{BG}$)*

$$\varrho_{in} \approx \sum_{\nu}\left[g^2\frac{12\Gamma(6)\zeta(6)}{\Gamma(4)\zeta(4)}\left(\frac{T}{T_{BG}}\right)^2\delta_{\nu L} + \frac{\hbar^2 v_F^2 \beta^2}{4a^2}\right] \times \frac{\Gamma(4)\zeta(4)(k_B T)^4}{e^2 \rho \hbar^4 v_F^2 v_\nu^5 k_F^3}$$

Efetov & Kim PRL.(2010)



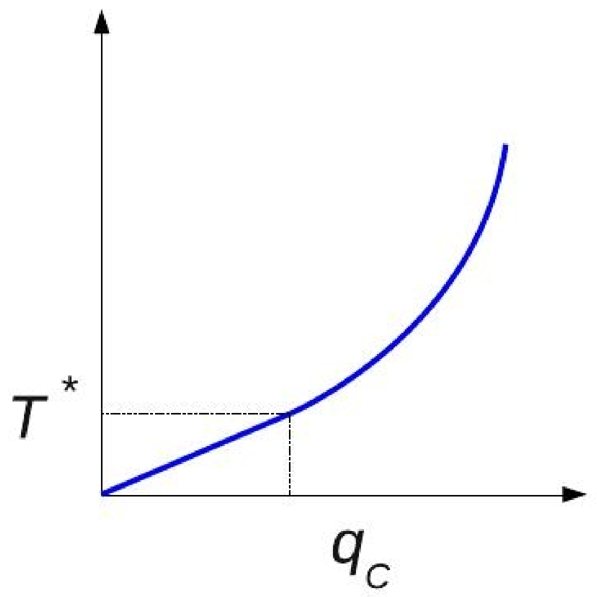
Mariani & von Oppen, PRB (2010)

arXiv: 1102.0807

When strain is present

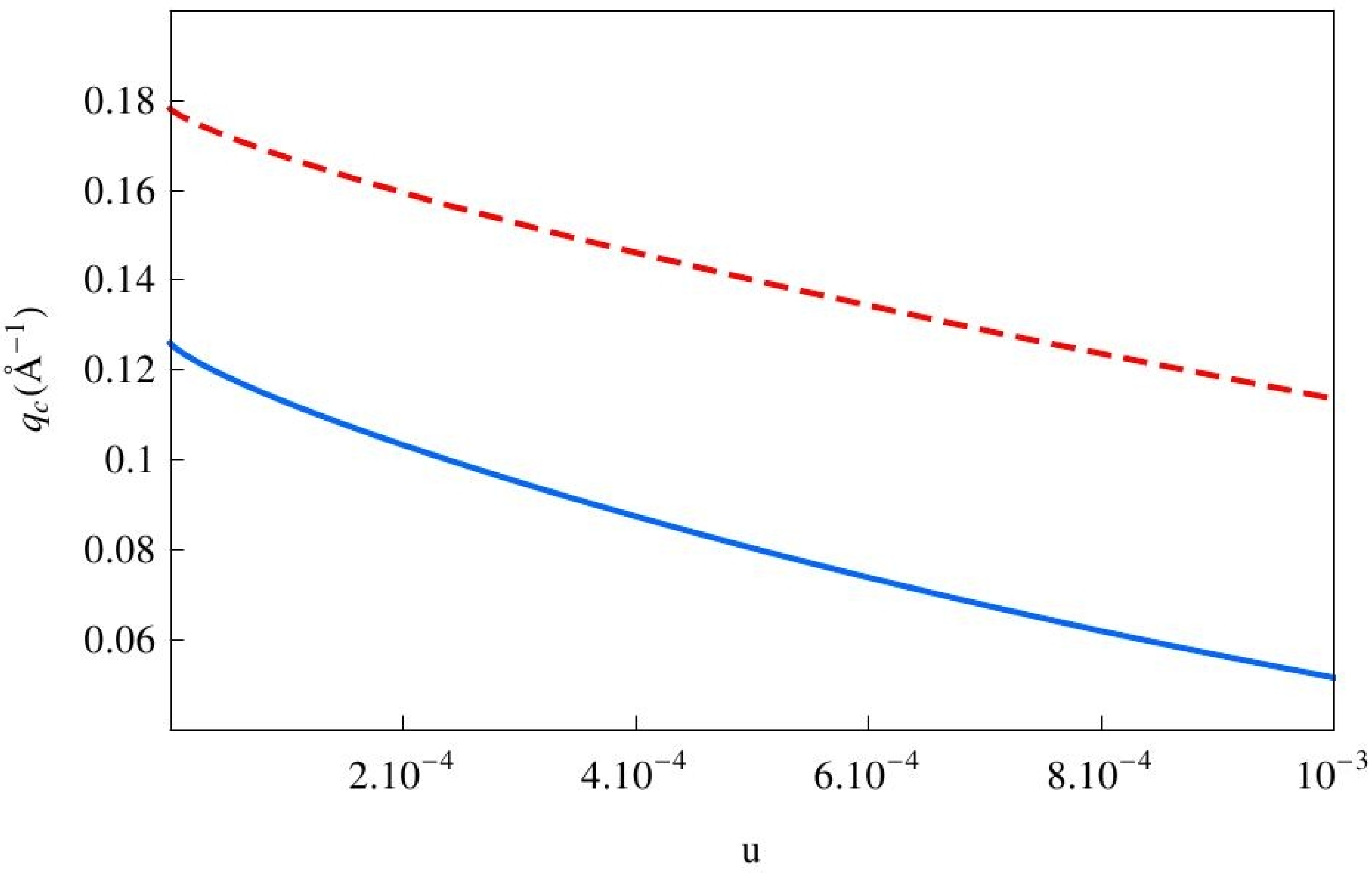
- Broken rotational invariance
  $\longrightarrow$ linear dispersion for flexural phonons

$$\omega_{\mathbf{q}}^F \simeq q\sqrt{\alpha^2 q^2 + \bar{u} v_L^2}$$



- Very low strain enters through cutoff under log
  - difference between samples due to different residual strain
- As strain increases $1/\tau$ decreases
  - density of low momentum phonons is strongly suppressed

Anharmonicity and strain



The bilayer graphene case

arXiv: 1102.0807

- *Electronically: massive chiral electrons*
- *Elastically: just a thicker membrane (ac. ph.)*